Exercise - the Cosmological Background Radiation

The cosmological background radiation is observed by using radio telescopes, and the observation shows that the radiation comes from all directions in almost equal intensity and energy distribution. The radiation is very well described as Planck radiation or black body radiation characterized by a single (absolute) radiation temperature T.
The temperature of the observed microwave background radiation (CMBR) is

(1) $T_{\text{CMBR}} = 2.7279\ \text{K}$ Present temperature of CMBR

If we draw the intensity vs wavelength graph for this Planck radiation, there is a maximum at the wavelength denoted by λ_{top} and there is the following connection to the radiation temperature T:

(2) $\lambda_{\text{top}} \cdot T = 2.90 \cdot 10^{-3}\ \text{m} \cdot \text{K}$ Wien's displacement law

Exercise 1: Calculate the wavelength of maximum intensity given the observed radiation temperature above.

Exercise 2: What was the temperature of CMBR at the time where the Universe was 100 times smaller? PS: All wavelengths – among them λ_{top}– scales with the size of the Universe.

The density of photons in the background radiation far exceeds the number of photons from other sources. Tætheden af fotoner i denne baggrundsstråling overstiger langt antal fotoner fra andre kilder. The density of photons in CMBR depends on the radiation temperature as given in the formula

(3) $n_{\text{photon}} = \frac{N}{V} = 16\pi \cdot \varsigma(3) \cdot \left(\frac{k_B \cdot T}{h \cdot c}\right)^3$ Photon density of CMBR

where ς is the Riemann zeta-function and $\varsigma(3) \approx 1.202$

N denotes the number of photons in the volume *V*, *c* is the speed of light in empty space, k_B denotes the Boltzmann constant and finally *h* is the Planck constant.

Exercise 3: Calculate the photon density of CMBR given the experimental value of the radiation temperature above

The density of photons (3) diminishes as the Universe expands the reason being the falling radiation temperature. Nevertheless the number of photons is a constant. This is the background for the following exercise:

Exercise 4: Imagine a time in the future where the temperature of CMBR is half of the present value. Answer in this context the following questions:

a) How many times will the photon density decrease compared to the present value according to (3)?

b) What has then happened to the wavelengths of CMBR according to (2)?
c) What has happened to the scale of the Universe – and the volume V in (3)?
d) Use your answers from above to argue that the number of photons N in (3) is unchanged.

The average number of protons and neutrons in the Universe is – according to data from the Planck satellite and other sources - 4% of 5.5 protons per cubic meter.

(4) $$n_{\text{nucleon}} = 0.04 \cdot 5.5\ \text{m}^{-3} = 0.22\ \text{m}^{-3}$$

Exercise 5: Calculate the average number of photons per nucleon in the Universe

The most nuclei in the Universe are in the form of protons as most matter in the Universe is hydrogen. At high temperature the hydrogen atoms will be ionized.
This temperature of ionization is in part determined by the energy of ionization of the hydrogen atom:

(5) $$E_{\text{ionization, H-atom}} = 13.6\ \text{eV}$$

The photons in CMBR have an average energy given by

(6) $$E_{\text{photon, average}} = 2.70 \cdot k_B \cdot T$$

Exercise 6: Calculate the temperature where $E_{\text{photon, average}} = E_{\text{ionization, H-atom}}$

In reality the temperature of ionization of the hydrogen atoms in the early Universe is only approximately 3000 K.

Exercise 7: Can you point to a reason why the temperature you calculated in ex. 6 is far too big?
Use the high value of the photon – nucleon ratio in your reasoning.
(Actually you have to use the Saha equation to make a more rigid reasoning)

Exercise 8: Evaluate the redshift of CMBR when you know it was created at a temperature of approximately 3000 K.