Cosmological redshift: Doppler shift upon Doppler shift upon…

As we shall see below the cosmological redshift can viewed upon as a series of Doppler shifts along the path of the light from emitter galaxy to observer galaxy. Or maybe more correct: the redshift occurs as a continuous Doppler shift experienced by the light as the next galaxy in the light path is also receding as the light is approaching.

We here looks upon a series of redshifts experienced by the light on its long way from emitter galaxy to observer galaxy.

The wavelength of the light as seen from the emitter galaxy we denote by the symbol λ_{em} and the wavelength of the same light as it is received by the observer galaxy we denote by λ_{obs}.

The wavelength as seen from the first galaxy in the light path after leaving the emitter galaxy we denote by λ_1, the wavelength as seen from the second galaxy we denote λ_2 etc. The wavelength as seen from the galaxy next to the observer galaxy we denote by λ_{n-1}. The wavelength of the light thus has been 'stretched' by Doppler shifts *n* times in total.

We now use the identity

$$\frac{\lambda_{obs}}{\lambda_{em}} = \frac{\lambda_{obs}}{\lambda_{n-1}} \cdot \frac{\lambda_{n-1}}{\lambda_{n-2}} \cdot \frac{\lambda_{n-2}}{\lambda_{n-3}} \cdot \ldots \cdot \frac{\lambda_2}{\lambda_1} \cdot \frac{\lambda_1}{\lambda_{em}}$$

In this equation we introduce the global redshift z and the local redshifts Δz_i:

$$1 + z = (1 + \Delta z_n) \cdot (1 + \Delta z_{n-1}) \cdot (1 + \Delta z_{n-2}) \cdot \ldots \cdot (1 + \Delta z_2) \cdot (1 + \Delta z_1)$$

If we assume the same value of all the local redshifts (Δz) we find

$$1 + z = (1 + \Delta z)^n \qquad \text{global and local redshift(s)}$$

And therefore

$$\ln(1 + z) = n \cdot \ln(1 + \Delta z)$$

where $\ln$ is the natural logaritm.

If the number *n* is a big number the local value Δz will be small. And we can use the approximation $\ln(1 + \Delta z) \approx \Delta z$.

We get:

$$\ln(1 + z) \approx n \cdot \Delta z$$

or

$$\Delta z \approx \frac{\ln(1+z)}{n} \qquad \text{local redshift}$$

As an example we assume a global redshift of 10 and divide it in 1000 parts, we get

$$\Delta z \approx \frac{\ln(1+10)}{1000} = 0{,}002398$$

This small value of the local redshift corresponds according to the (non-relativistic) Doppler law to a relative velocity of

$$v = \Delta z \cdot c = 0{,}002398 \cdot 300000\,\mathrm{km/s} = 719\,\mathrm{km/s}$$

This is indeed a non-relativistic velocity.

If the galaxies do not change velocity (the Milne Model) the emitter galaxy will have the velocity

$$v_{em} = 719 \cdot 1000\,\mathrm{km/s} = 719\ 000\,\mathrm{km/s} = 2{,}398\ c$$

where c is the (local) speed of light $c = 300\ 000\ \mathrm{km/s}$.

In the Milne model (the empty Universe) we can translate from general relativistic coordinates to special relativistic coordinates. And for velocities we translate as follows:

$$v_{em,\ SR} = c \cdot \tanh(2{,}398) = 0{,}9836\ c$$

It is thus clear that the emitter galaxy is not moving faster than light!

The reason for this presumably 'faster than light' velocity is the distance definition. The distance to the galaxies is a sum of locally measured proper distances at rest relative to the nearby galaxies on the long light path from emitter galaxy to observer galaxy. The distances thus are not Lorentz shortened as they would be in special relativity. That is why there is no upper limit to the galaxy velocity when we use general relativistic coordinates (distance and time).

In the Milne Model we can translate all velocities to special relativistic values, and therefore we can use a special relativistic formula for the redshift:

$$1 + z = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} = \sqrt{\frac{1+0{,}9836}{1-0{,}9836}} = 11{,}00$$

Corresponding to the redshift 10 – the value we assumed in the beginning of this example.

In the Milne Model the redshift can be viewed upon as a single Doppler shift no matter how big the value of the redshift. But it can – as described above – also be seen as a series of minor Doppler shifts. Or perhaps more correct: a continuos Doppler stretch of the wavelength as the light moves through the expanding Universe.