Measuring the gamma activity of the Mini-Generator

- An alternative way of using the cesium/barium mini-generator…

The so-called mini-generator (cesium/barium) contains a certain amount of the radioactive isotope Cs-137. The Cs-atom decays to Ba-137 in a beta-minus transition, leaving the daughter nucleus in the excited state Ba-137*. This excited state emits a gamma photon in the decay to the ground state. It is a fraction of these gamma photons we are measuring using a GM-tube.

The half-life of Cs-137 generator is approximately 30 years. Therefore we might well assume a constant rate of creation of the radioactive nucleus Ba-137* in the short time of the experiment (10 minutes or so).

The beta-decay process is

$$^{137}_{55}Cs \rightarrow \, ^{137}_{56}Ba^* + \, ^{0}_{-1}e + \bar{\nu}_e \qquad \text{equation for the beta-decay of Cs-137}$$

where one of the neutrons in the Cs-137 nucleus is decaying to a proton, an electron $^{0}_{-1}e$ and an electron-antineutrino $\bar{\nu}_e$.
The beta-decay of the Cs-137 nucleus also can create the Ba-137 nucleus directly in its ground state, but this decay does not create a gamma-photon and is not relevant in this exercise.

The decay of Ba137*- state is described by the equation

$$^{137}_{56}Ba^* \rightarrow \, ^{137}_{56}Ba + \gamma \qquad \text{equation for the gamma decay of Ba-137*}$$

The GM-tube measures a small fraction of the photons (γ).

If the Mini generator is left alone for a while the rate of gamma decays of Ba-137* inside the generator will match the rate of creation of new Ba-137* nuclei. There is then an equilibrium-activity of gamma decays.

The radioactive decay law of Ba-137* tells us

$$A = \lambda \cdot N \qquad \text{the radioactive decay law}$$

where N is the number of radioactive Ba-137*-nuclei in the generator, og A is the activity of these nuclei. And λ is the decay constant of Ba-137*.

At the beginning of the exercise we are ’milking’ the generator which means that we force an eluting solution through the mini-generator using a syringe leaving the Cs-137 in the genertor. We remove as much of the radioactive Ba-137* from the generator as possible by eluting the generator a few times. The activity of the remaining Ba-137* in the generator will therefore be lower than before the milking. But the number of newly created Ba-137* nuclei per second is unchanged as the amount of Cs-137 in the generator is unchanged (the radioactive decay law).

Designating the equilibrium activity of the generator A_{eq}, the equation for the rate of change in Ba-137* nuclei in the generator will be

$$\frac{dN}{dt} = A_{eq} - \lambda \cdot N \qquad \text{rate of change in Ba-137* - number}$$

because A_{eq} is the number of *newly created* Ba137*-nuclei per unit time, and $\lambda \cdot N$ is the number of *decayed* Ba-137*-nuclei per unit time. The difference must therefore tell us the rate of change in the number of Ba-137* per unit time.

If we choose to express this growth equation solely by the activity A we get the equation

$$\frac{1}{\lambda}\frac{dA}{dt} = A_{eq} - A$$

or

$$\frac{dA}{dt} = \lambda \cdot (A_{eq} - A)$$ Differential equation for the activity of Ba-137*

The solution to this equation is

$$A(t) = A_{eq} - (A_{eq} - A_0) \cdot e^{-\lambda \cdot t}$$ Solution formula for the activity of Ba-137*

where the activity at the beginning of the measurements is designated A_0.

(you can make the substitution $u = A_{eq} - A$ in the differential equation above if you want to prove this formula)

As the formula shows its possible from the measurements to determine both A_{eq} og the decay constant λ.

The half-life of Ba-137*-decay is then calculated using the equation

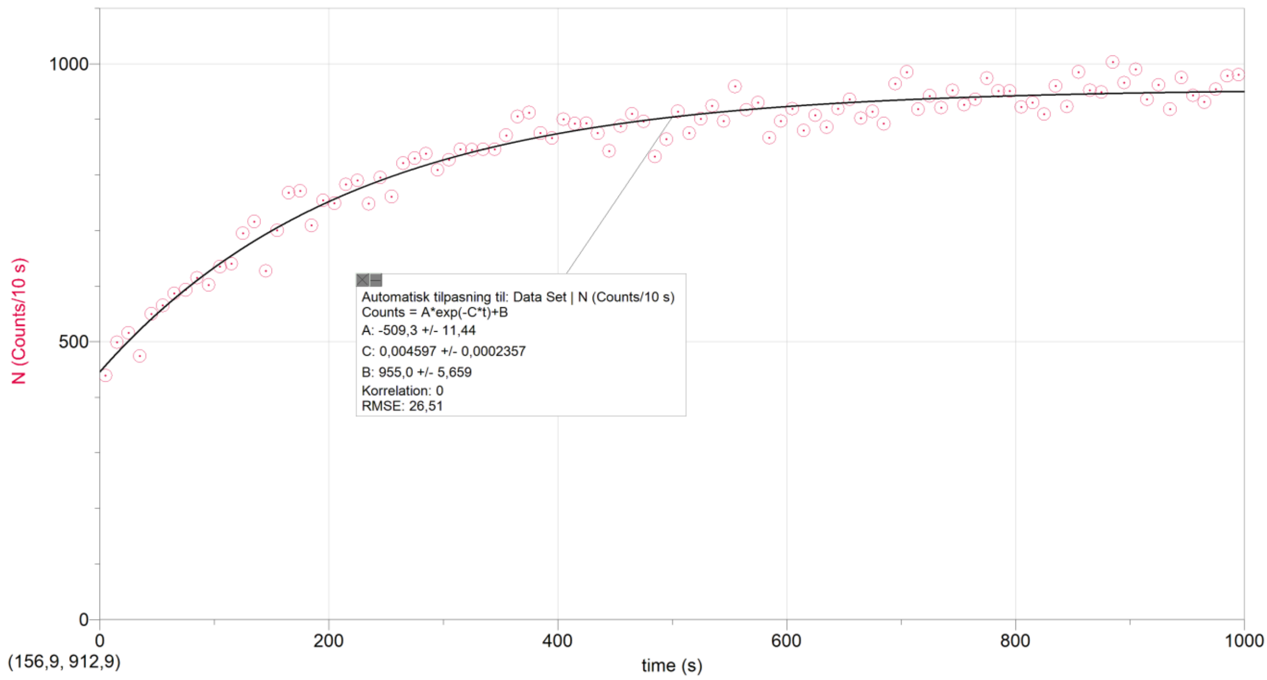
$$T_{\frac{1}{2}} = \frac{\ln(2)}{\lambda}$$ Half-life and decay constant

Accepted value is

$$T_{\frac{1}{2}} = 2{,}552\ \mathrm{min} = 153{,}2\ \mathrm{sec}$$ Accepted value, half-life of Ba-137*

If your counting rate is too small to get good results you can try to use a GM-tube having a bigger ‘window’ or use ex 2 GM-tubes in parallel.

Below a measurement on a mini-generator and analyses using the program LoggerPro.



Automatisk tilpasning til: Data Set | N (Counts/10 s)
Counts = A*exp(-C*t)+B
A: -509,3 +/- 11,44
C: 0,004597 +/- 0,0002357
B: 955,0 +/- 5,659
Korrelation: 0
RMSE: 26,51

Figure: activity of mini-generator vs time – and analyses by LoggerPro

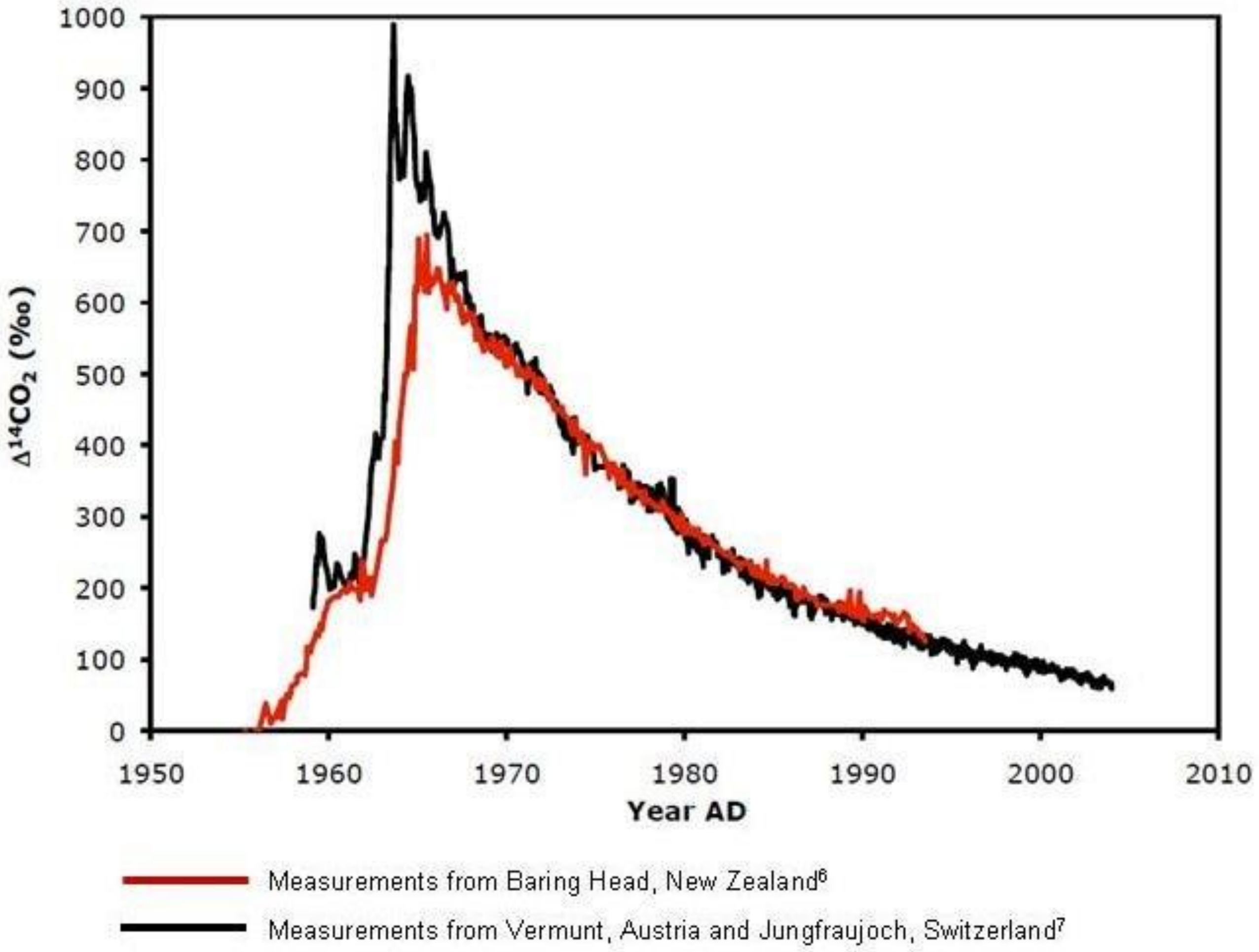
As shown the equilibrium activity is 955/s and the decay constant is 0.004597/s +/-0.0002357/s, giving a half-life of 151 sec +/- 8 sec. Not so bad! The initial activity is 446/s. Thus we have eluated more than half of the radioactive Ba-137 nuclei before the beginning of experiment.

I reality the activity of the mini-generator is app. 10 mikrocurie, or approximately 370 kBq for a newly bought generator. The measured activity is therefore only a small part of the actual activity.

Another equilibrium /non-equilibrium case for radioactive isotope

As seen above it's the physical half-life of Ba-137* that determines how quickly the equilibrium is restored. After a few physical half-lives the generator is 'up and running' again. This is not always the case. In the earth' atmosphere there is continuously created new C-14 atoms by cosmic radiation, and continuously C-14 atoms are removed from the atmosphere. And these two may be in equilibrium – but not always as we shall see below.

The newly created C-14 atoms will react with the oxygen of the atmosphere creating CO_2 – molecules. These molecules will – as times go by – be absorbed in the ocean or enter in the biology associated with the atmosphere. Therefore the physical half-life of C-14 (5730 years) is not relevant in restoring equilibrium of C-14 in the atmosphere. The figure below shows the result on C-14 in the atmosphere of nuclear atmospheric testing in the beginning of the 1960s. The biggest H-bomb ever exploded has left its own mark on this curve (The Tsar – bomba 30. oktober 1961). The red curve is from the southern hemisphere whereas the black curve is from the northern hemisphere, where most of the testing were made.



Source: https://www.esrl.noaa.gov/gmd/ccgg/isotopes/bombspike.html

The enhanced activity of C-14 diminishes exponentially after the tests were forbidden (Nuclear Test Ban Treaty October 1963). And diminishes by app. 4% per year. This corresponds to a half-life of 18 years. Very much less than the physical half-life.