Supermassive Black Hole
in the Center of the
Milky Way

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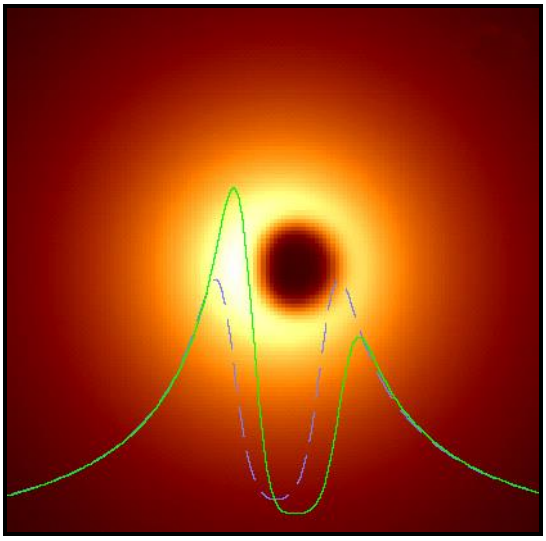

(figure from Falcke et al. 1999, ref. 28)

The supermassive black Hole in the Center of the Milky Way

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1. Introduction
Stars and gas-clouds near many galaxy-centers show great velocity-dispersions, and this fact is taken as an indication of the existence of supermassive (i.e. masses in the range from millions to billions of solar masses) black holes. The gravitation from these massive black holes is the reason for the large velocities of the stars and clouds near the galaxy centers. Also the long radiojets seen emanating from many galaxy-centers are usually taken as a sign of the presence of massive, rotating black holes, the rotational-axes assumed to be the direction of the dual jets. Black holes are considered to be the central powerhouse in actice galaxies (AGN).
In several cases the velocities of the stars nearest to the center of the galaxies exceeds 1000 km/s – nevertheless the distance from the stars to the proposed black hole is often so great, that Newtonian mechanics is certainly good enough as a first approximation – and allows a determination of the gravitational mass of the central mass – no matter the nature of this. If the velocities or the velocity-dispersions shows a Keplerian signature (vel. proportional to $r^{-0.5}$ where r is the distance to the center), a mass of the central object can be determined.
The central mass observed in the center of The Milky Way coinciding with the radio-source Sagittarius A* gives the best possibilities to study how a black hole behaves – if it is indeed a black hole. The discovery of this radio-source was done in the year 1974 by Balick and Brown at the NRAO-interferometer at Green Bank (ref.11). Only future observations, possibly VLB interfero-meter-observations – will tell us the details of the beast, for example the direct observation of gravitational lensing of NIR-radiation or radio waves coming from behind the heavy central mass or maybe a black shadow showing directly the black hole. This will probably be possible in a few years.
This report will describe parts of the current understanding of the massive central mass and the derivation of the mass and the possible angular momentum of the object. And try to answer the question: is it really a black hole?

*2. The apparent proper motion of SgrA**
The proper motion of the radio-source SgrA* has been measured in radiowaves with respect to background extragalactic reference frame (ref.17). The result is:

$$\mu_{l,*} = -6.18 \pm 0.19 \text{ mas/y}$$
$$\mu_{b,*} = -0.65 \pm 0.17 \text{ mas/y}$$

where (l*,b*) is the galactic coordinates of SgrA*. It can be seen that the main part of the motion is along the galactic plane. When combined with measurements in NIR (ref. 18), it can be shown that almost all of this apparent proper motion can be ascribed to the motion of the Sun - where the Sun participates in the differential rotation of the Galaxy (220 km/s) and has its own peculiar motion relative to the local standard of rest. The z-component of the Solar peculiar motion relative to the local standard of rest is 7.2 +/– 0.4 km/s. The tangential velocity of the Sun in the galactic plane is 20 km/s. The assumed distance to the galactic center is 8,0 kpc, as confirmed by the astrometric measurements of the stellar orbit of the star S2. After removing this Solar motion from the measurements, it is shown that SgrA* moves with 5 +/–3 km/s perpendicular to the galactic plane. The motion of SgrA* in the galactic plane is more insecure – the reason being the relative insecurity of the local standard of rest in the galactic plane (10 – 20 km/s). This relative slow motion of SgrA* relative to the galactic center confirms that SgrA* is the dynamical center of the Galaxy.
Measurements of the proper motion of SiO-stars (stars associated with SiO – maser emission) at both radio and infrared wavelength in the central cluster shows that the central star cluster moves

with SgrA* within 40 km/s per coordinate-axes, or 70 km/s for the 3-dimensional motion. This is small compared to the spacevelocity of S2 which exceed 5000 km/s at the pericenter of its orbit. The radioposition of SgrA* is within 10 mas of the focus (graviational center) of the S2-elliptical orbit.
Thus there seems to be good evidence for the postulate that SgrA* is very close to the dynamical center of the Milky Way - or is coincident with the center. If the central mass is a massive black hole, the radio and infrared source are expected to be very close to the hole, probably within 10 Schwarzschild-radii (associated with an accretion disk or maybe a jet).

3. Determination of the Mass of the Central dark Object
The most direct way to determine the mass of the central object in the Milky Way-galaxy is to observe stellar orbits generated by the gravity of this object. And then use Kelpers 3. law on these stellar orbits to determine the mass of the central object.

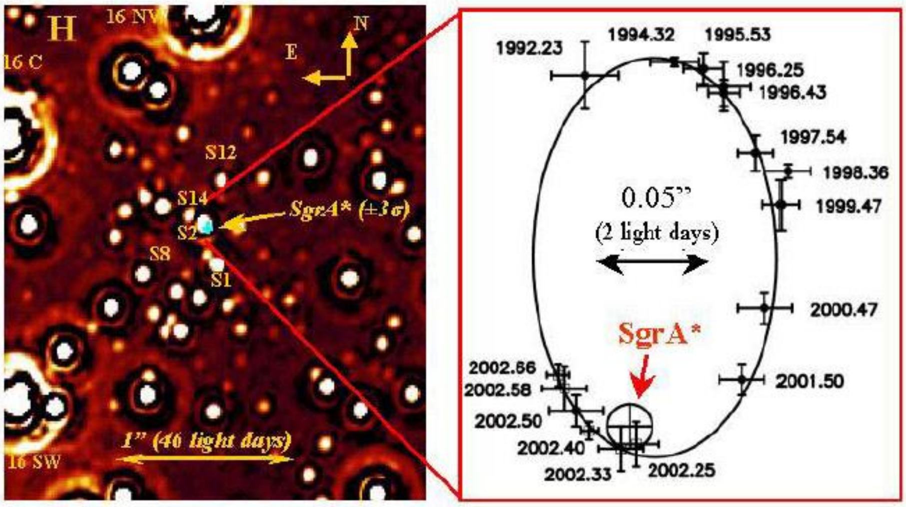

Fig. Fejl! Ukendt argument for parameter.**: 10 years of Observations of the Orbit of the Star S2. The figure has been taken from from Schödel et al. 2002, 2003**

In fig. 1 we see on the left inset a picture of the most central parts of the Milky Way in NIR. The center is completely blocked in visible light because of the vast amounts of dust and gas in the line of sight (lying in the galactic plane) towards the center as seen from the Earth. However, in radio, NIR and X-rays it is possible to penetrate this barrier.
The scale of the picture is shown, the width is approximately 2''.
The picture was taken using the NAOS/CONICA camera/adaptive optics instrument on UT4 on the VLT (40 mas resolution). The Radiosource SgrA* is marked with an arrow and colored blue.
The right inset shows the orbital data and best Keplerian fit of the orbit of S2 around SgrA* (circle with cross). The positions from 1992 to 2001 are measured by the NTT-telescopes SHARP-camera, whereas the positions in 2002 are measured by the NAOS/CONICA instrument on UT4. The speed of the star reached over 5000 km/s in 2002.
The analysis of the stellar orbit gives the following parameters (Schödel et al. 2002, ref.1):

Table 1: Derived orbital parameters for the star S2

Parameter	*Value*	*Formal error(1 σ)*	*Astrometric error*
Mass of black hole M ($10^6 M_{Sun}$)	3.7	1.0	1.1
Period P (years)	15.2	0.6	0.8

Time of pericentre passage (year)	2002.30	0.01	0.05
Eccentricity *e*	0.87	0.01	0.03
Angle of line of node (degrees)	36	5	8
Inclination *i* (degrees)	+/– 46	3	3
Angle of node to pericenter (degrees)	250	4	3
Semi-major axis *a* (mpc)	4.62	0.39	0.15
Separation at pericenter r_{min} (mpc)	0.60	0.07	0.15

The formal errors stems from the orbital fits, the astrometric errors are due to the 10 mas astrometric uncertainty. The distance to the galactic centre is assumed to be 8 kpc. The angle of the line of nodes is measured anticlocwise relative to the direction North on the figure. The angle from node to pericenter is measured from the node in the north-east quadrant in the direction of motion of S2. The sign of the inclination-angle is not known, because no line-of-sight-motion are used in the analysis (these measurements has only been possible later).
The semi-major axis projected on the sky would be 0,119'', and therefore

(1) $$a = 0.119'' \cdot 8\ \text{kpc} = 952\,\text{AU} = 5.5\,\text{light days} = 1.42 \cdot 10^{14}\,\text{m} = 0.00461\,\text{pc}$$

in accordance with the values given in table 1. And the mass of the black hole follows easily (Keplers 3. law):

(2) $$M = \frac{a^3}{P^2} = \frac{952^3}{15.2^2} M_{\text{Sun}} = 3.7 \cdot 10^6 M_{\text{Sun}}$$

This method is the most direct for the determination of the mass of the black hole, therefore there is great hope, that the incrising resolution and sensivity of the measurements with the NAOS-/CONICA instrument will make it possible to measure even orbits of faint starts closer to SgrA* in the coming month and years. Infrared interferometry using the VLT, the Keck and the Large Binocular Telescope will give even better resolution, down to a few mas – making it possible to study relativistic motion close to the black hole.
The Schwarzschild-radius of the hole (assuming it to be non-rotating) is given by

(3) $$r_g = \frac{2GM}{c^2} = \frac{M}{M_{\text{Sun}}} \cdot 2.95\,\text{km} = 3.7 \cdot 10^6 \cdot 2.95\,\text{km} = 10.9 \cdot 10^6\ \text{km} = 0.073\,\text{AU}$$

Here *G* is the gravitational constant, *c* is the vacuum speed of light. The star S2 does not come close to the Schwarzschild-radius of the hole, actually the closest approach is

(4) $$r_{\min} = \frac{r_{\min}}{r_g} \cdot r_g = \frac{0.0006 \cdot 3.09 \cdot 10^{13}\text{km}}{10.9 \cdot 10^6\ \text{km}} \cdot r_g = 1700\, r_g$$

this is far from the point where relativistic effects on the orbit will be visible within the time of a few orbital periods – and the tidal effects on the star itself will also be small (the mass of the star is approximately 15 $M_{Sun,}$ the radius app. 7 R_{Sun}). It is of course easily shown, that r-min also can be written as

(5) $$r_{\min} = 124\,\text{AU} = 17\,\text{lighthours} = 0.00060\,\text{pc}$$

If the infrared interferometric technic can give a resolution of let's say 1 mas, we can study motions as close as

(6) $$r = 0.001'' \cdot 8\text{kpc} = 8\,\text{AU} = 110\,r_g = 1{,}1\,\text{lighthours}$$

At this distance the period P (measured by a distant observer) for circular motion around the black hole is given by

(7) $$P = 2\pi \cdot \sqrt{\frac{2r^3}{c^2 r_g}} = 2\pi \cdot \sqrt{\frac{2\cdot(110 r_g)^3}{c^2 r_g}} = 1631\frac{r_g}{c} = 1631 \cdot 36\,\text{s} = 16{,}5\,\text{h}$$

The speed (relative to the speed of light) in this orbit is

(8) $$\beta = \frac{1}{\sqrt{2(r/r_g - 1)}} = \frac{1}{\sqrt{2(110-2)}} = 0.068$$

implying that motion of the star is almost in the relativistic regime.
It should be noted, that the expression (7) is valid both in Newtonian gravitational physics (Keplers 3. law), but is also valid for circular motion in the Schwartzhild-metric. The expression (8) is valid in the Schwartzhild-metric. The speed is here defined as length per unit proper time of an observer at rest at the point where the motion happens. Had we used Newtonian gravitational physics, and divided the speed by the velocity of light c, the result would be 0.067 – very close to the result (8).

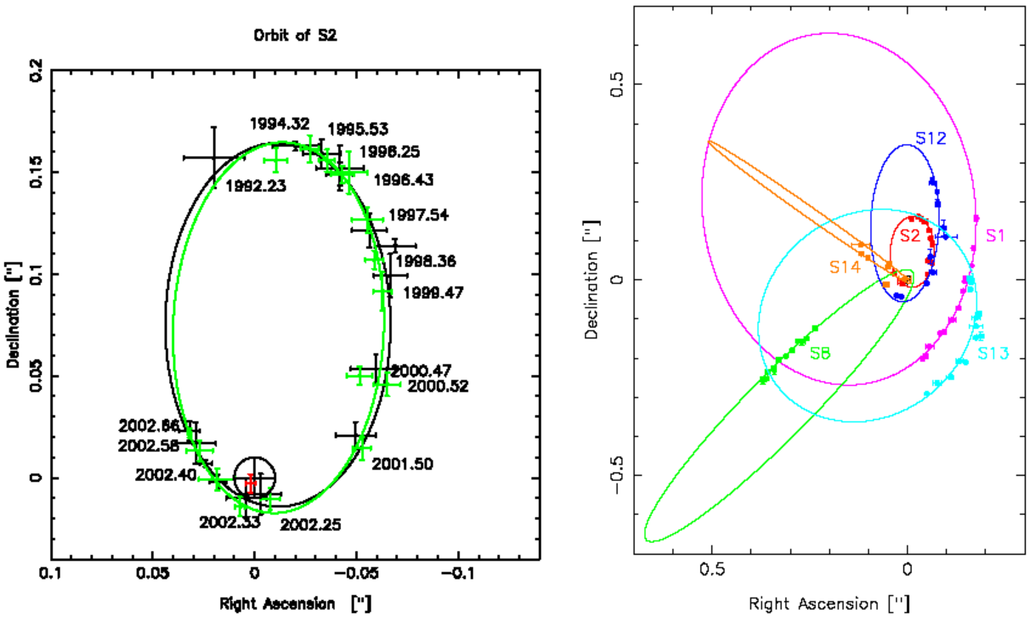

Fig. 2: orbits of fast-moving stars near SgrA* - see ref. 9

Not strange, as we are still relative far from the black hole.
If there are stars that close to the black hole, it will be possible to follow such stars through many orbital periods, and relativistic effects such as the advance of the pericenter of the orbit (if the orbit is not a circular orbit, of course!!), the dragging of inertial frames in case of a rotating hole, the asymmetric movement of light around a rotating black hole etc.
We might ask the question: how close can a star come to the black hole before it is disrupted by tidal forces from the hole? A rough estimate can be calculated using the formula (ref. 19)

$$r_{\text{tidal disruption}} = R_* \cdot \sqrt[3]{\frac{M}{M_*}}$$

If we use the estimated mass and radius of the star S2 we get

$$r_{\text{tidal disruption}} = R_* \cdot \sqrt[3]{\frac{M}{M_*}} = 7 \cdot R_{Sun} \sqrt[3]{\frac{3.7 \cdot 10^6}{15}} = 439\, R_{Sun} = 28\, r_g$$

thus the star S2 is not at all close to the limit where it will be destroyed by tidal forces, as the closest approach r_{min} is 1700 r_g.
It should be mentioned, that the orbits for other stars has been measured as well. See fig. 2. However, the orbital parameters of these (S1, S8, S12, S13, S14) are not yet precise enough to give a much more precise determination of the enclosed mass compared to the orbital parameters of S2. The green orbit for S2 are due to a new analysis of the data for S2 – where also the focus of the orbit was taken as free parameters – giving the red cross as result. As can be seen on the figure the position of this is well inside the black circle giving SgrA* position determined by radio-metric measurements(+⁄– 10 mas). Future astrometric mea-surements of the orbits will no doubt give a more precise value for the enclosed mass. The analysis above gives the estimate $3.4 \pm 0.5 \cdot 10^6\, M_{Sun}$

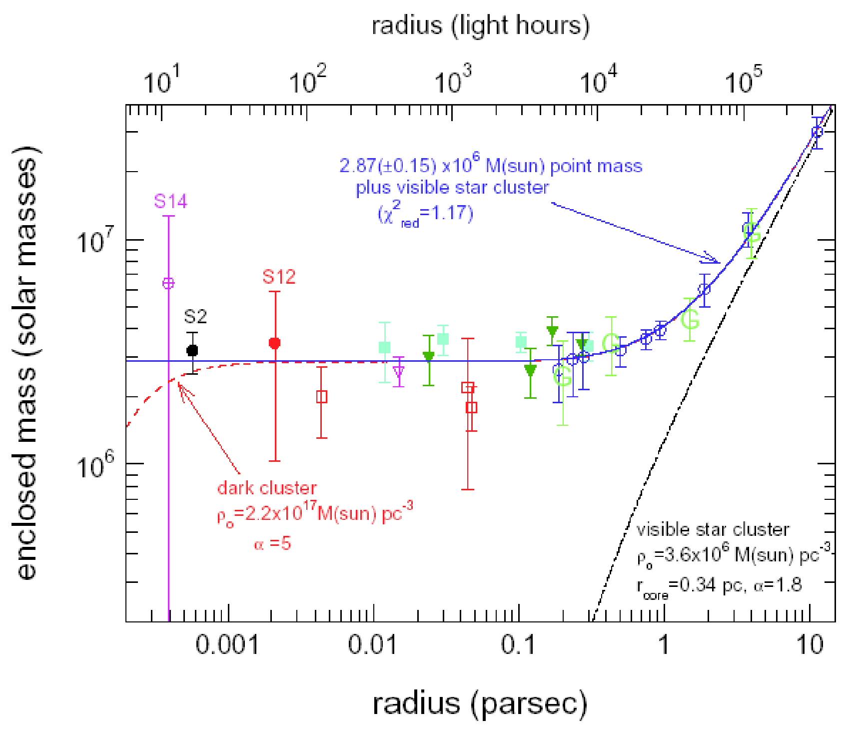

Fig. 3: enclosed mass measured from the galactic center (ref. 9)

for the enclosed mass. In accordance with the value given in table 1. No radial motion observations are included in the analysis. If such observations are made, it will be possible to make an analysis using a as a free parameter – giving a measure of the orbit independent of the distance to the galactic center. And of course also giving a value for this galactic center distance.
Actually measurements of precisely this kind have already been made! (ref. 15 and 16). Four measurements of the radial motion of the star S2 have been made in 2002.4177, 2002.4205, 2003.21 and 2003.35 (using H I Br-γ 2.1661 μm and He I 2.1126 μm lines). With these absolute values of velocities corrected for the motion of the earth relative to the galactic center it is possible to analyse the orbit of the star without reference to the galactic center distance – but it is as mentioned above also possible to derive the galactic center distance from the analysis. The sign of the inclination-angle of the orbit can also be found. And it shows that the star S2 is behind the focus of the ellipse at pericenter. Thus it is rotating against the general galactic rotation. This fact taken together with the early spectral type (08 – B0 main sequence star, mass in the range 15 – 20 solar masses) gives an age of the star of not more than 10 mio. years – and makes it diffucult to understand how the star formed in a region with strong tidal forces. And the young age gives only a short time for migration from bigger distances toward the center.
The derived new parameters can be seen in table 2. It should be noted that the mass at the focus of the ellipse is not a fit-parameter, but is derived from the 3. law of Kepler (as in eq. (2)). The galactic center distance R_0 is taken as a free parameter in the fit. The position of the focus of the ellipse is given in a commen infrared astrometric frame – as opposed to the radiometric frame. There is an uncertainty between the infrared and the radio astrometric frame of +/– 10 mas.

Table 2: Derived orbital parameters for the star S2 (ref. 16)

Parameter	*Value*	*Uncertainty*
Mass of black hole M ($10^6 M_{Sun}$)	3.65	+/– 0.25
Period P (years)	15.559	+/– 0.337
Time of pericentre passage (year)	2002.339	+/– 0.011
Eccentricity e	0.880	+/– 0.006
Angle of line of nodes (degrees)	45.3	+/– 1.5
Inclination i (degrees)	- 47.9	+/– 1.3
Angle of node to pericenter (degrees)	245.1	+/– 1.6
Semi-major axis a (mas)	0.1200	+/– 0.0026
Position of focus of elipse x_0 (mas)	2.2	+/– 1.2
Position of focus of elipse y_0 (mas)	- 3.2	+/– 1.1
R_0 galactic center distance (kpc)	7.99	+/– 0.38

With improved orbital elements for other stars than S2 it will be possible to determine the mass of the central object and the distance to the galactic center with even better precision – possibly making the distance to the center of the galaxy the best known distance in the cosmic distance ladder. It should be noted that the value R_0 is in good agreement with most recent distance-estimates.

The enclosed mass as a function of the distance to the galactic center can be seen in fig. 3 (ref. 9). It can be seen on this figure that the data are consistent with a central point-mass with a mass (blue curve) of $2.87 \pm 0.15 \cdot 10^6 M_{Sun}$. And that there is quite ”emty” space from the central mass out to the distance 0.2 pc. The estimate from the modelparameters used in making the blue graph tells us, that there is at most a few hundred Solar masses inside the pericenter of S2 – stars from the central cluster of stars surrounding the galactic center – apart, of course from the central point-mass. Calculating the sky-projected velocity-dispersions (from proper-motion measurements and the galactic center distance) of stars at different sky-projected distances from SgrA* , these velocity-dispersions follow a Keplerian signature, being proportional to $r^{-0.5}$ where r is the projected distance to SgrA*. The movements of the stars therefore seems dominated by one central mass – the socalled enclosed mass - as long the projected distance to SgrA* is below 0.1 pc (ref.36).
We will not go into many more details of mass-determination of the central cluster summarized on figure 3. But the pericenter of the star S2 is also here seen to be important in the analysis. The enclosed mass stays constant downto at least this pericenter-distance. The pericenter of the star S14 are closer to the central mass than the pericenter of S2. But the precision of the orbital elements are not nearly as good as for S2.

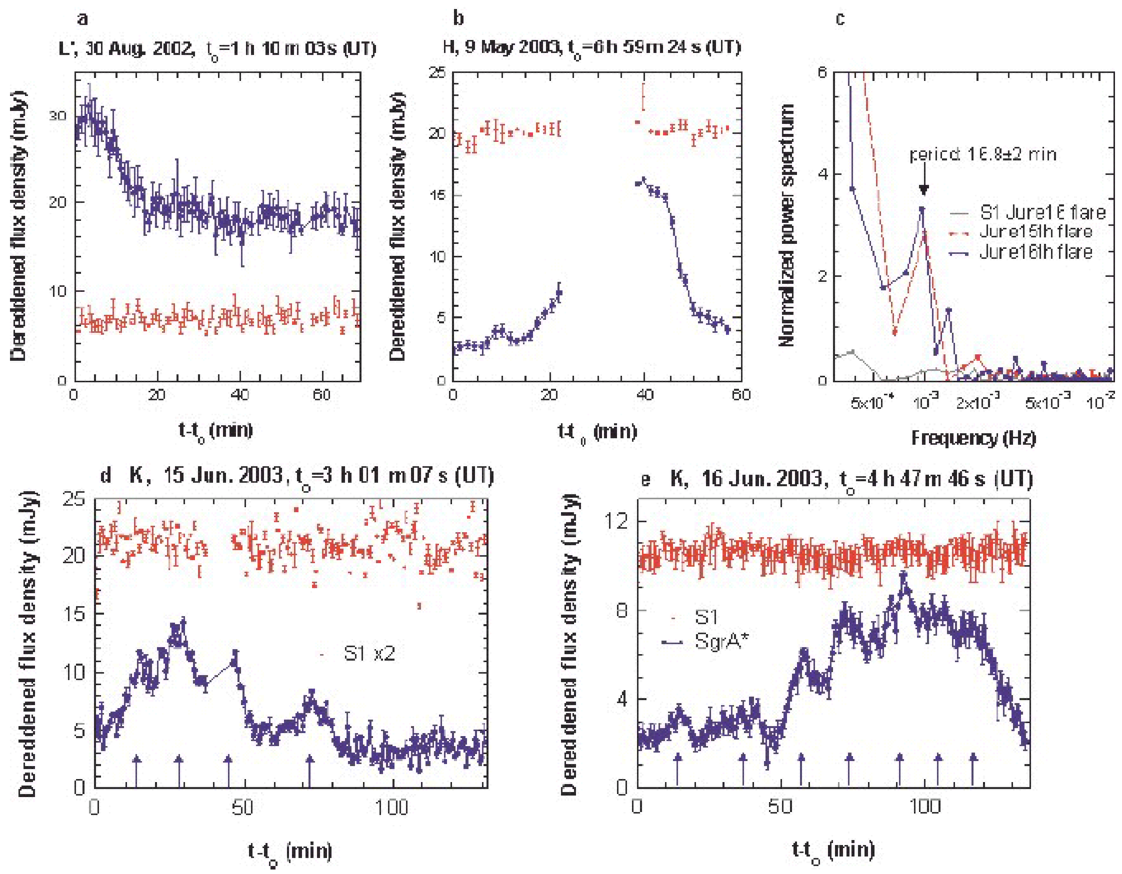

Fig. 4: NIR light-curves for SgrA*-flares(VLT), blue curves, ref. 10. SgrA*-data are the filled blue circles – red datapoints are lightcurves for the nearby star S1. Time is relative to UT-time listed above each graph. The blue-graph power-spectrum shows a peak at a period of 16.8 +/– 2 min. in the SgrA*-flux.

4. Determination of the angular momentum of the black hole
It can be shown, that not all circular orbits in the Schwarzschild -metric are stable – as opposed to the case of Newtonian theory. The innermost stable circular orbit (ISCO) in the Schwartzhild-metric is located at the *r*-value

(9) $$r_{ISCO} = 3r_g$$

and the period of this orbit (as measured by a distant observer) is given by the expression (7):

(10) $$P_{ISCO} = 2\pi \cdot \sqrt{\frac{2r_{ISCO}^3}{c^2 r_g}} = 2\pi \cdot \sqrt{\frac{2 \cdot (3r_g)^3}{c^2 r_g}} = 2\pi\sqrt{54}\,\frac{r_g}{c} = 46.2 \cdot 36\,\mathrm{s} = 27.7\ \mathrm{min}$$

The speed given by (8) is 50% of the speed of light!
Thus, if we receive periodic signals from regions close to the black hole, its is expected, that the shortest period from orbital motion of gas in a accreation-disk should be approximately 28 min.
In fig.4 we see lightcurves of flares in the near infrared coming from an area very close to the source SgrA* (ref.3,10) .
A period of approximately 17 min can be seen in the power-spectrum. If this signal is understood as radiation from gas circulating the black hole, the period is clearly too short compared to the 28 min., calculated above for a non-rotating black hole with the mass $3.7 \cdot 10^6$ solar masses.
This could be a sign of a rotating black hole, if this period is understood as the period of the innermost stable circular orbit.
But how fast should the black hole rotate to give this period of 17 min?
To answer this question we must turn to another metric (other than the Schwarzschild), namely the Kerr-metric for a rotating black hole.
This metric is more complicated than the Schwarzschild-metric, see app. 1 for some details.
The important thing here is the period of circular motion, especially the innermost stable circular orbit.
Circular motion in the equatorial-plane is possible, if the angular velocity $\omega = d\varphi/dt$ satisfies the following equation:

(11) $$\Gamma^r_{\varphi\varphi}\omega^2 + 2\Gamma^r_{t\varphi}\omega + \Gamma^r_{tt} = 0$$

where the angle φ is the rotation-angle around the symmetry-axes, t is the time as measured by a distant observer. The Γ-symbols are the Christoffel-indices which can be calculated from the metric given in appendix 1.
The result is

(12) $$\frac{\partial g_{\varphi\varphi}}{\partial r}\omega^2 + 2\frac{\partial g_{t\varphi}}{\partial r}\omega + \frac{\partial g_{tt}}{\partial r} = 0$$

and if we use the metric from app.1, we get

(13) $$\frac{2r^3 - a^2}{r^2}\omega^2 + 2\frac{a}{r^2}\omega - \frac{1}{r^2} = 0$$

The solutions of this equation is

(14) $$\omega_{\pm} = \frac{\pm 1}{r\sqrt{2r} \pm a}$$

where the plus-sign should be used for rotation in the same direction as the hole (co-rotation), and the minus-sign for the opposite direction. The unit for ω in (14) is c/r_g .

The radial parameter *r* is measured in units of r_g as given by (3), and the rotational parameter *a* (not to be confused with the semi-major axes of the star S2!!). *a* is the angular momentum per unit mass of the hole divided by *c*, and is measured in units of r_g .

However, stable circular orbits are not possible for all values of *r* (see e.g. James B. Hartle, 2003 p.316).

The criterion of stability against small changes in the *r*-value gives a connection between *a* and *r*, we will here limit ourselves to the co-rotation-solution.

It is not difficult to show (using the ref. above), that you have to solve the following equation to get the innermost stable circular orbital radial parameter (units: r_g for *r* and *a*):

(15) $$r = \sqrt{3r - \frac{a^2}{r}} - a \cdot \sqrt{3 - \frac{1}{r}}$$ eq. for r_{ISCO}

Solving this eq. gives the functional dependence between *r* and *a* and we can insert the values in (14) and finally find the relation between the period *P* and the rotational parameter *a*.

(16) $$P = \frac{2\pi}{\omega_+}$$

We will here limit ourselves to numerical solutions, see app. 2, 3. The numerical solution was done by the author (of this report). The relation between *a* and *P* is of course valid for all rotating Kerr-black holes – taking into account the appropriate units. What you should know are the mass of the black hole (used to calculate r_g) and the period *P* for the innermost stable circular orbit in the direction of the rotation of the hole. This will typically be seen in measurements of the intensity of the radiation from an outbreak stemming from gas falling into the black hole. The gas will circulate the hole and loose angular momentum through friction and other mecanisms. When we have got the period then we can use the graph to read of the value of rotational *a* – in units of r_g . But now back to the actual case:

To use the (*a*,*P*) – graph we must know the value of *P*. The unit for this is r_g/c and this was already calculated in (7) to 36 sec.

(17) $$\frac{P}{r_g/c} = \frac{17\,\text{min.}}{36\,\text{s}} = 28$$

Using the (a,P)-graph and the (a,r)-graph we finally find

(18) $a = 0.25 r_g \quad r = 2.1 r_g$

and this is 50% of the maximum-value for the angular momentum per unit mass. A very fast rotation. And it is likely that the effective generation of NIR happens at a little outside the innermost stable circular orbit (see ref.10) – therefore the value of a in (18) is probably a minimum value. (Warning: in many (most?) texts the unit of a is taken to be only $\frac{1}{2} r_g$).

If we take a lower limit of the mass of the hole: $3.4 - 0.5 \cdot 10^6 \, M_{Sun}$ we can again calculate r_g and P from eq. (17), and using appendix 3 we get a minimum value of a : $a_{\min} = 0.15 \; r_g$.

The rotational period of the hole itself is given by (units for a and ω as mentioned above)

(19) $$\omega_{hole} = \frac{a}{r} = \frac{a}{\frac{1}{2} + \sqrt{\left(\frac{1}{2}\right) - a^2}}$$

In this formula, the parameter r is the radial parameter of the event-horizon in the equatorial-plane of the hole.

Plugging in the value of a given by (18), we get ω_{hole} = 0.27. The corresponding period is 14 min, analogous to (16). And not the 30 seconds, reported elsewhere (ref.6).

It should be noted however, that it would be nice to see this 17 min. period in more measurements in NIR, and maybe also in X-ray-intensity measurements before we can be sure that this period represents the period of the innermost stable orbit. (See comments below)

Actually it has now been shown, that there is a connection between flares in NIR and X-ray, see ref. 7. In this reference, the radiation is described as due to heated and accelerated electrons, and the fast flares due to syncrotron-radiation, all stemming from the accretion-disk near the black hole. This would suggest a high degree of polarization of the radiation, as it is indeed seen in NIR.

A relevant question to the above analysis would be: could there not be other processes that could generate a period of 17 min.?

And the answer is probably no! Other processes that could generate periodic variations in the intensity of the NIR radiation are acoustic waves in a thin disk, orbital node precession of the acretion disk (Lense-Thirring-precession). However these are expected to be too slow to explain the observed period (see again ref. 10).

According to *Aschenbach et al.* 2004 (ref.35) XMM and Chandra-measurements of flares in X-ray actually shows a set of periodic or quasiperiodic oscillations, being in agreement with the periods seen in NIR. These periods (seen in at least two spectra) are approximately 100s, 219s, 700s, 1150s and 2250s. The 1150s period is tentative being identified as the NIR-period. Aschenbach notes that there are four cyclic gravitational modes associated with a black hole accretion-disk, namely the Kepler-frequency (eq. (14)), the vertical and radial disk-perturbation frequenciesand finally the Lense-Thirring frequency equal to the difference between the Kepler- and the vertical-perturbation frequency. If the following identifications are made (the only combination that gives a consistent determination of the black hole mass and rotational parameter):

- 219s-period: Kepler-period at the innermost stable circular orbit
- 692s-period: Vertical-perturbation-period at the innermost stable circular orbit

- 1117s-period: Radial-perturbation-period at the r-value giving the minimum radial-perturbation-period

then the Lense-Thirring-period is 320s – one of the other periods found in the NIR-powerspectrum of fig.4. Using the relations between the different periods/frequencies (expressed by the mass *M* and the rotational parameter *a* of the black hole and the value of the radial parameter *r* where the oscillation occurs) it is possible to predict the black hole mass and the rotational parameter of the black hole:

$$M = \left(2.72 + 0.12 / - 0.19\right) \cdot 10^{6}\, M_{\mathrm{Sun}}$$

$$a = 0.4970 \quad + 0.0012 / - 0.0037$$

Thus the black hole is rotating allmost as fast as possible (max-value of a is 0.5). And the mass is somewhat lower than the value given in table 2.
It should be noted however, that it has not been shown that all four frequencies are expected to show up as a physical frequency in the power-spectrum – and the identifications of the measured spectral-periods to the Kepler-period, vertical- and radial-periods – taken together with the the value of *r* where the oscillation occurs - seem somewhat arbitrary. More measurements of strong flares in X-ray and NIR – and a better timeresolution in NIR – are probably needed to resolve these questions.

5. What is the real size of the central dark object of the Milky Way?
Let us assume, that the central object is a black hole. In that case one might think, that the Schwarzschild-radius (or diameter) would give the size of the dark area, we might expect to see - if our telescopes had the right angular resolution. But that is not entirely correct. We must remember that gravity from the hole will bend lightrays and in some cases absorb lightrays/photons. Given the Schwarzschild-metric its possible to show, that the black hole has a circular absorption-cross-section for photons (or extremely relativistic particles) given by

(20) $$\sigma_{absorption} = \pi \cdot \frac{27}{4} r_g^2$$ *absorption-cross-section for photons*

It is here assumed that the hole is non-rotating. This corresponds to an impactparameter for the photon of

(21) $$b = \frac{3\sqrt{3}}{2} r_g \approx 2.60 \cdot r_g$$

A photon with this impact-parameter will enter a (unstable) circular orbit around the black hole. Is the impactparameter smaller the photon will approach the event-horizon and never be seen again. So given a background of some stars or other sources of electromagnetic radiation behind the black hole, we will see a dark circle with a diameter of 2*b*, or - if we put in the numbers (see (3)):

(22) *diameter of dark circle* = 2*b* = 5.20 r_g = 0.38 A.U.

Seen from the earth the angular resolution required to dissolve this is

(23) $$\textit{minimum required angular resolution} = \frac{2b}{r_{GC}} = \frac{0.38\,\mathrm{AU}}{8000\,\mathrm{pc}} = 47\,\mu as$$

It should be noted that this size is a shadow-size – up against the radiation coming from behind the black hole – not the radiation from an acretion-disk or a jet which might also extend to the front of the hole, depending on the unknown orientation of the rotational-axes of the black hole.
It might seem impossible ever to reach this kind of resolution – but how close are we today?
New VLBA-observations at 43 GHz (7 mm wavelength) show, that the radio-emission from the central object comes from a region of size 1.8 AU (see ref. 13). This corresponds to 24 +/– 2 times the Schwarzschild-radius of the hole.
This is only 5 times greater than the expected value for diameter of the black circle given by (21) – so we are actually very close to seeing the beast - if it is a black hole!
At greater wavelength the radio-source seems much bigger – a fact that can be explained by the ”fog” that the radiowaves has to pass in their way to the telescope. This ”fog” is due to scattering by turbulent interstellar plasma along the line of sight. – and has nothing to do with the size of the source. This scattering obeys a power-law where the size of the source grows proportional to the square of the wavelength. With the latest measurements using smaller wavelength however, the size of the source is greater than what would be expected from the scattering-law-scaling – and therefore the *intrinsic size* of the source can be derived. The longest baseline used in the experiment was 2000 km.

The values of the following table is taken from ref. 13.

Table 3: Intrinsic Size of the Major Axis of SgrA*

Wavelenght (cm)	Measured Size (μas)	Scattering Size (μas)	Intrinsic Size (r_g)
1.35	2635 +37/-24	2533 +20/-20	72 +15/-11
0.69	712 +4/-3	669 +5/-5	24 +2/-2
0.35	180 +20/-20	173 +2/-2	6 +5/-5

It can be seen that the intrinsic size is wavelenght-dependent. This imposes constraints on the models that should explain the sources of the radiation.
If we assume that the mass of the central object is confined within 24 Schwarzschild-radii we get for the average mass-density

(24) $$\rho = \frac{M}{V} = \frac{3\cdot 10^6\,M_{Sun}}{\frac{4}{3}\pi\cdot(24\cdot 0.073\mathrm{AU})^3} = 1.3\cdot 10^5\,M_{Sun}/\mathrm{AU}^3 = 1.1\cdot 10^{21}\,M_{Sun}/\mathrm{pc}^3$$

The dynamical lifetime of a cluster of objects with this density against internal collissions or evaporation would be less than 1000 years – thereby excluding this possibility, making Sgr A* the most convincing existing case for a massive black hole (ref. 14). See later for a discussion of the nature of the central dark object and the dynamical lifetime of a cluster.

6. Radiation from the central massive object

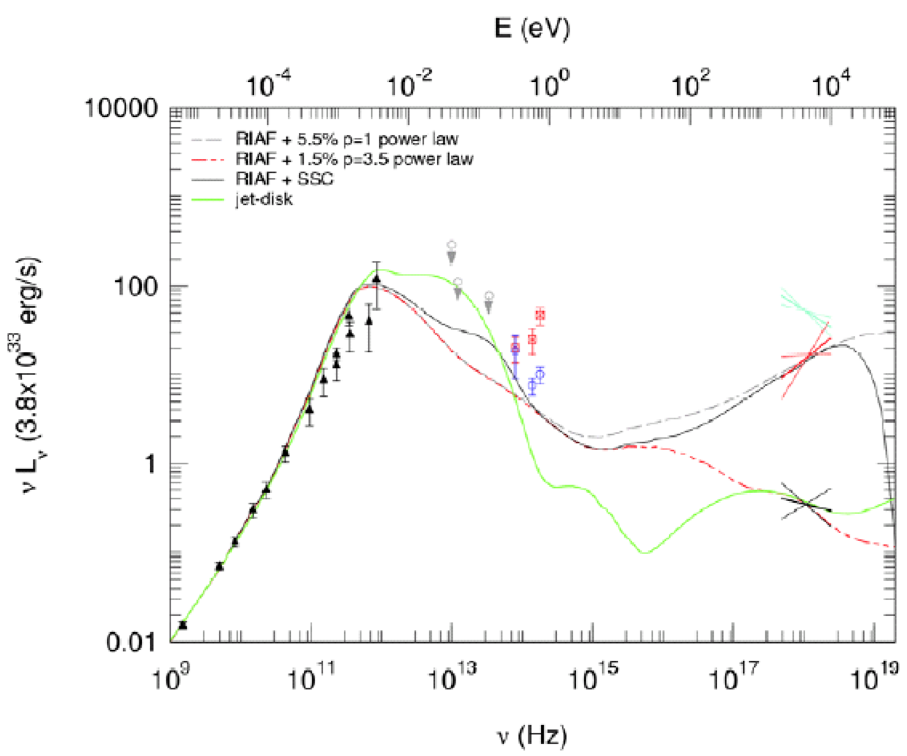

Fig. 5: the electromagnetic spectrum of SgrA* (ref. 20)

Measurements of the electromagnetic radiation associated with SgrA* are summarized in fig.5. Here radio-, (upper limits to) IR, NIR and X-ray-measurements are displayed. The observed flux-density S_ν has been multiplied by the frequency ν. And the luminosity L_ν is calculated using the formula

$$L_\nu = 4\pi \cdot D^2 \cdot S_\nu$$

where D is the distance to the center of the Milky Way (here taken to be 7.94 kpc)- thus assuming spherical symmetry. The measurements has been corrected for extinction and absorption. Error-bars are +/– one standarddeviation. The black triangles denotes the quiscent (that is: 'normal', slowly varying) radio-spectrum of SgrA*. Open grey circles denotes upper limits to the IR luminosity. The three X-ray data-sets are: black denotes the quiscent state (measured by Chandra X-ray Telescope), red denotes the flare seen fall 2000 (XMM) and the light-blue denotes the fall 2002 flare (XMM). Open red squares marks the NIR peak-emission, observed in four flares – open blue circles marks the deredded H, K_S and L' – luminosities in the quiescent state.

A prominent feature of the spectrum in fig.5 is the small amount of luminosity above 10^{13} Hz (note that the luminosity has been multiplied by the frequency).

The flares in NIR and X-ray lasts 30 – 40 minutes and is seen approxemately simultaneously – therefore probably requiering a common physical cause (ref. 22). The flares in NIR happens at timescales from 10(!) – 100 minutes. Also the variability in X-ray (up to 50 times) is bigger than in NIR. The short period in the flares indicate that the origin of these are close to the central object – in the case of a black hole scenario close to the innermost stable circular orbit. X-ray flares occur on average once a day. The quiscent radiation seems to come from a more extended region (1 arcsec). Linear polarization in the sub-mm range have been observed.

As argued above, the central object of the Milky Way is very compact – probably a black hole. It is therefore natural to try to explain the origin of the electromagnetic radiation from this object using models with a black hole in the center.

Before we enter a few details of the models of fig.5 it is suitable to introduce the socalled Eddington-limit. It is defined by equality between gravitational force and radiation-pressure force from the radiating object (see e.g. ref. 21):

(25) $$L_{\text{Edd}} = \frac{4\pi \cdot c \cdot G \cdot M \cdot m_{\text{p}}}{\sigma_{\text{T}}} = 1.3 \cdot 10^{38} \cdot \frac{M}{M_{\text{Sun}}} \text{ erg/s}$$

where M is the mass of the black hole, m_p is the mass of the proton (associated with the electron!) and σ_T is the Thomson cross-section for scattering of photons on electrons.
In the case of SgrA* we get

(26) $$L_{\text{Edd}} = 1.3 \cdot 10^{38} \cdot \frac{3.65 \cdot 10^{6} M_{\text{Sun}}}{M_{\text{Sun}}} \text{ erg/s} = 4.7 \cdot 10^{44} \text{ erg/s}$$

In fig.5 several such models have been applied to the data, and the various curves shows varying degrees of succes in explaining the observations.
The abbreviation RIAF means *Radiative Inefficient Accretion Flow* – a model for the accretion-flow and the emission from the source. The luminocity from the source is small in this model:

(27) $$L \le 10^{-8} L_{\text{Edd}}$$

In the case of SgrA* the luminosity is a factor of 3 less than this limit. The reason for the low luminosity could be the small accretion-rate, maybe as small as $10^{-5} M_{\text{Sun}} / \text{year}$. Another reason in the RIAF-model is that the model is inefficient in converting the lost gravitational energy to radiation. The RIAF –models describes a hot quasi-spherical rotating accretion flow with viscosity. The radiation is created by a thermal electron population and electrons having a nonthermal power law-spectrum. The non-thermal electrons are being accelerated by shocks or magnetic reconnection. For the electrons in the non-thermal state a percentage and a power *p* (numberdensity of electrons proportional to the gammafactor of these in the power of *minus p*) is given in the figure for the different models.
The abbreviation SSC means *Syncrotron Self-Compton* radiation. Low-energy photons are Comton-scattered by relativistic electrons to higher energies. This process can explain parts of the X-ray-emission in the flares. Another part could be pure syncrotron-radiation of the accelerated electrons. It is not clear whether the SSC-effect is needed or whether pure syncrotron-radiation from electrons accelerated in shocks or magnetic reconnection (like in the Solar flares) can explain the flare-observations. Or whether syncrotron-radiation from jet-accelerated electrons contributes to the energy-spectrum.
The reason why these flares are visible is probably that the accretion-rate of the black hole is very small. At higher accretion-rates (using these RIAF-models) these flares are ’buried’ in the quiescent emission.
The source of the gas that is accreted on to the black hole is probably mass-loss from a cluster of stars 10 arcsec from the hole, including blue supergiants - being dominated by IRS 13E which is 3.5 arcsec from Sgr A* on the sky. The interaction of these stellar winds shocks the gas and heats it to temperatures where it emits X-rays. The total mass-loss-rate for these stars is 10^{-3} M_{Sun}/year. This is much higher than the accretion-rate of the central black hole, the main part probably being thermally driven out from the center in stellar winds(ref. 23). The stars in the central cluster seen in fig. 2 , right inset are probably main sequence-stars with much lower mass-loss-rate. Spectroscopy of one of the stars, S2, suggests that it is a main sequence O/B star, as already noted. The hot gas

from the stars can be the source of the diffuse X-ray-emission in the central parsec as seen by the Chandra X-ray telescope.

7. Is the SgrA-source a black hole?*
Now to the million-dollar question: is there a black hole in the center of the Milky Way? And how can we be sure?
Several models have been proposed as alternatives to a black hole.
One reason to invent such models is to try to avoid the singularity of the black hole.
Another reason for inventing some of these models is that the Universe seem to contain large amounts of dark matter that only manifests itself only by gravitational forces – actually the main part of the matter is dark! And it has to be somewhere. So why not also in the galactic centers?
Several of these supermassive central objects emits very little electromagnetic radiation if compared to the Eddington-limit – they are rather dark.
We will here concentrate on the following alternatives to the black hole scenario:

a) A cluster of non-luminous objects such as brown dwarfs or stellar remnants
b) A supermassive star of fermions such as neutrinoes
c) A supermassive star of bosons

A cluster of non-luminous objects such as brown dwarfs or stellar remnants
Is it possible to put some dynamical constraints on these models? The answer is yes. The reason is that such collections of many objects gravitationally bound to each other has a finite probability of either evaporate or to collide and form heavier objects (ref. 24).
If we assume a Plummer-model of mass M (giving the least centrally concentrated model for a cluster with a given mass because it has the steepest falloff of the density observed in any astrophysical system):

(28) $$\rho(r) = \rho_0 \cdot \left(1 + \frac{r^2}{r_c^2}\right)^{-\frac{5}{2}}$$

where ρ_0 is the central density, $\rho_0 = \dfrac{3M}{4\pi \cdot r_c^3}$ and r_c is the core-radius. It proofs to be useful to replace the two parameters ρ_0 and r_c by the cluster half-mass and its half-mass-density ρ_h (the mean density within the clusters halfmass-radius R_h) of the Plummer-model:

(29) $$R_\mathrm{h} = 1.3 r_\mathrm{c} \qquad \rho_0 = 4.4 \rho_\mathrm{h}$$

The evaporation-lifetime against weak gravitational scattering of a cluster of mass M consisting of (identical) objects with mass m_* can be shown to be (ref. 24)

(30) $$t_\mathrm{evap} = \frac{4.3 \cdot 10^4 \cdot (M_\mathrm{h}/m_*)}{\ln[0.8 \cdot (M_\mathrm{h}/m_*)]} \cdot \left(\frac{\rho_\mathrm{h}}{10^8 M_\mathrm{Sun}/\mathrm{pc}^3}\right)^{-1/2} \mathrm{yr}$$

Here the quantity M_h is half of the mass of the cluster. The other limit to the lifetime of the cluster comes from the collision time, here used in a Plummer model and applying the velocity-dispersion of a Plummer-model (the collision time is the characteristic timescale for each star to collide with another, taking gravitational focusing into account):

(31) $$t_{\mathrm{coll}} = \left[23.8 G^{1/2} M_{\mathrm{h}}^{1/3} \rho_{\mathrm{h}}^{7/6} \left(\frac{r_*^{\,2}}{m_*} \right) \cdot \left(1 + \frac{m_*}{2^{1/2} \cdot \rho_{\mathrm{h}}^{1/3} \cdot M_{\mathrm{h}}^{2/3} \cdot r_*} \right) \right]^{-1}$$

If we take the example given in eq. (24) and assume that the half-density is given by this value, the mass of the objects is assumed to be 1.4 Solar masses, the radius of the star 10 km (a neutron star), we get from eq. (30) and (31):

(32) $t_{\mathrm{evap}} = 1250\,\mathrm{yr}$ and $t_{\mathrm{coll}} = 930\,\mathrm{yr}$

Taking the minimum of these two, the age is clearly inconsistent with the fact that we see the object today – unless we are in a very special period of the Universe. We must assume that the cluster has existed for a substantial part of the lifetime of the Galaxy – that is 10 Gyr. Thus we can safely(!) conclude that the central object does not consist of neutron-stars!

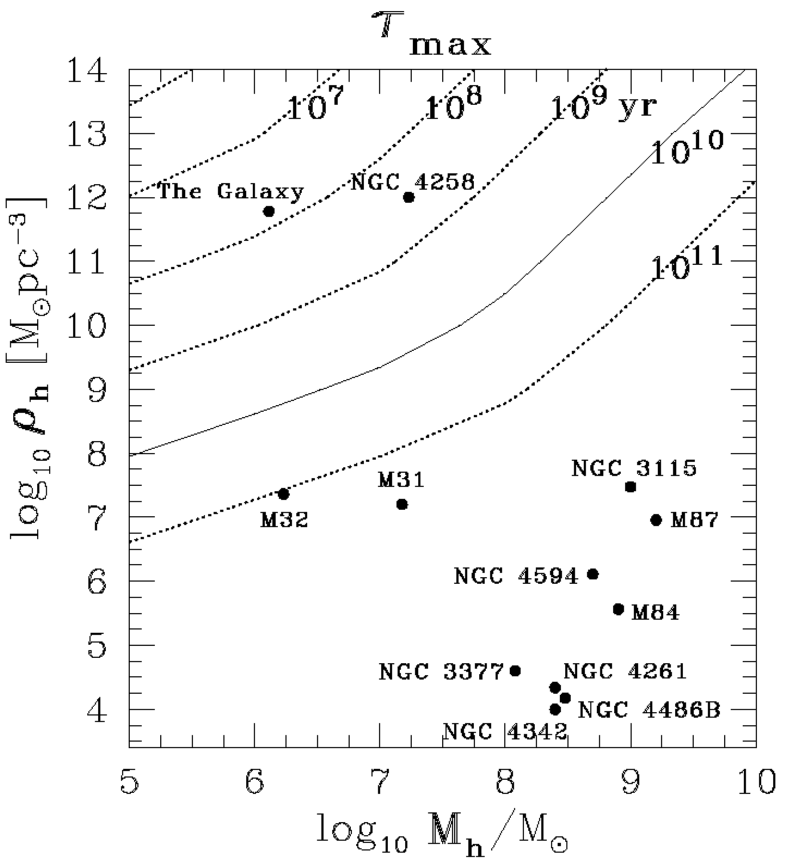

Fig. 6: lifetimes of central clusters of galaxies

In ref. 24 the focus has been on exisiting astronomical objects. That is black holes with mass > 3 M_{Sun}, neutron stars with $1.4\,M_{\mathrm{Sun}} \le m_* \le 3\,M_{\mathrm{Sun}}$, low mass objects (e.g. planets) with $m_* < 3 \cdot 10^{-3} M_{\mathrm{Sun}}$ supported by the pressure of atoms, objects with masses in the range $3 \cdot 10^{-3} M_{\mathrm{Sun}} \le m_* \le 1.4 M_{\mathrm{Sun}}$ supported by electron-degeneracy pressure such as white dwarfs, brown dwarfs (up to 0.09 M_{Sun}). The radius of the objects has also to be known to calculate the collision-time (31). These mass-radius-relations can be found in the reference 24.

The authors then calculate the minimum of the two times (30) and (31) for a given half-mass and half-density (but with varying astronomical objects). The resulting *maximum* (for different astronomical objects)lifetime τ_{max} of the cluster can be seen in fig. 6.

The half-density of the central mass of the Milky Way is calculated using the minimum distance of the star nearest to SgrA* in 1997. Today we have a better limit on the size of the central mass. If we use not the value given by eq. (24), but the closest approach of the star S2 to SgrA* as R_h, we get

(33) $$\rho_h = \frac{M_h}{\frac{4}{3}\pi \cdot R_h^3} = 2 \cdot 10^{15}\, M_{Sun}/\mathrm{pc}^3$$

a value that falls outside the top of the figure but gives an estimated lifetime of the central mass in the Milky Way of less than 1 mio. years – clearly an unrealistic short value. This leads to the conclusion that the central mass cannot consist of any known sort of astronomical objects. But what other objects will give a lifetime of the cluster that is comparable to the lifetime of the Galaxy? Actually there is the possibility of small dark holes. If we in the formula (30) set the evaporation-time equal to 10 Gyr and also use the matterdensity (33), we find $m_* = 0.000075 M_{Sun}$. Therefore the cluster could persist of small black holes (Schwarzschild-radius less than 22 cm!) with mass smaller than this limit. The value (33) – where R_h is equal to the minimum distance of the star S2 to the central object – indicates that only half of the mass of the cluster is inside. This is in contradiction to the enclosed mass of fig.3 – therefore it would be safe to use a smaller value for R_h and thereby getting at higher value of the half-density, as displayed for the model used in fig.3 (the red dashed curve). The mass of the small black holes making up the cluster will therefore have to be even smaller than $m_* = 0.000075 M_{Sun}$.
These black holes are not the end-products of stellar evolution. But could be primordial – created in an inflatory Big Bang.
These *dynamical* considerations leads to the conclusion that only in NGC 4258 and the Galaxy we can exclude known astronomical objects as being the only constitutients of the central mass. However there are other models of the central mass that avoids the black hole paradigm.

A supermassive star of nonbaryonic fermions (such as neutrinoes)
This alternative to the black hole scenario has no singularity or event-horizon, consisting of a ball of selfgraviting nonbaryonic fermions. These objects of elementary particles may have formed in the early Universe during a first order gravitational phase transition (ref.26). The ball of fermions 'fights' gravity by the degeneracy-pressure of the constituing particles.
The massive central object is composed of selfgraviting degenerate neutrinos (or more generally nonbaryonic selfgraviting degenerate fermions). The mass of this neutrino can – if the mass of the central massive object of M87 (which has been determined to $3 \cdot 10^8 M_{Sun}$) is to be expained in this model and at the same time is the most massive (Oppenheimer-Volkoff-limit) possible object of this kind – be determined to be 15 keV (ref.25). This gives a radius of the neutrinoball in M87 of 4.5 r_g (r_g is the gravitational radius of M87), therefore the dynamics of objects orbiting the central mass is for greater radii very much the same as in a black hole scenario.
However, in galaxies with massive central dark masses much less than this, the fermion-ball will be considerably greater measured in units of the the gravitational radius if we use the same mass for the neutrino. In the Milky Way the radius of the fermion-ball will be 21 light-days – much greater than the distance from the pericenter of the star S2 to the gravitational center of the galaxy (only 17 lighthours). Therefore the orbit of this star will be considerably influenced as compared to the

scenario of a central massive black hole. Only a minor part of the mass of the neutrino-ball will be inside the pericenter of S2 and the orbit will not be an ellipse (Kepler-orbit), as the amount of mass contained in a ball with radius equal to the distance between the star and the center will vary in the elongated orbit. And the enclosed mass felt by the star will be smaller than the enclosed mass of stars orbiting further away from the center. This is not the case as can be seen in fig.3.
However, the radius of the fermion-ball can be made smaller by choosing a bigger neutrino-mass. Therefore we cannot use this argument to exclude the fermion-ball from the game. But then the model cannot be used in the M87-case and we have to find another model for M87.
The tidal forces in this model will – because of extended size of the ball (which has its root in the Fermi-exclusion-principle and therefore the existence of a Fermi-energy) be rather small. Stars are therefore not subjected to tidal disruption in this model – in contradiction to at least one very recent observation (RXJ1242-11 – see ref. 27).
An accretion disk will in the inner parts of the ball move at approximately constant velocity and cannot therefore generate the X-ray-flares that has been seen coming from the galactic center. A way of avoiding this difficulty is to invent a neutronstar near the center of the ball – and the gas falling down on the surface of this is then made responsable for the flares (ref. 23) – as it has been observed in several other neutronstars.
The mass of the neutrino has to be at least 50 keV – if the neutrinoball should have a radius less than the pericenter-distance of S2. Therefore the neutrino cannot be one of the 3 known species (electron, muon, tau-neutrino). This would make make the total massdensity of the Universe bigger than the critical value. It must therefore be e.g. a sterile neutrino, an axion or a gravitino.
The ball of neutrinos will – as opposed to the black hole – be able to transmit electromagnetic radiation even right through the center. The ball will act as a gravitational lense (magnifying glass) making the star-velocities behind (relative to the observer) seem greater. This transparency to EM radiation could serve as a way to observationally distinguish between a black hole and the weakly interacting dark matter alternatives without a singularity.
Another serious problem in this neutrinoball-scenario is: what happens to the gas, stars and stellar remnants that is being accreted onto the ball – how can it be avoided that this matter will fall to the center of the neutrino-ball and form a massive black hole?
These problems has led to the conclusion that the non-baryonic fermion-ball scenario is not very plausible ('bad standing').

A supermassive star of bosons

In this alternative model the dark matter consists of elementary particles in the form of bosons. Like the former alternative to the black hole scenario this model has no singularity or event-horizon. There is not a Fermi-energy as the exclusion-principle is not relevant for bosons. The reason why the ball does not collapse is in this case the Heisenberg uncertainty-principle. The size of the boson-ball is not much greater than the Schwarzschild-radius, and therefore most of the predictions of this boson-model are hard to distinguish from the black hole scenario (ref.26).
The particles that are supposed to make up these boson-balls could be the Higgs-boson , the axion or Goldstone boson. None of these has yet been found in nature. The creation of the boson-stars could again happen in the early Universe in a first-order gravitational phase transition – the mass of the boson being connected the time of decoupling of the boson from the thermal pool. A greater mass means an earlier decoupling.
Almost all of the predictions of a black hole scenario in relation to particles, stars etc. moving in the external gravitational field can be made by this model too, because the extension of the very compact boson-star is not much greater than the black hole (that is, a few times the gravitational radius of a black hole of the same mass). The maximum velocity of a circular orbit is approximately

30% of the speed of light – not much different from the black hole where the velocity in the innermost stable circular orbit is 50% of the speed of light (in the Schwartzschild-case). However, the model is completely transparent for EM radiation, as it is composed of particles only interacting by gravity. As in the former model no dark circel-area will exist. And the ball may act as a gravitational lense (magnifying glass). Also, particles can move right throught the center without being captured.
But we must again ask the question: how can it be avioded that gas, stars, stellar black holes etc. being accreted by the boson-ball will not fall to the center of the gravitational potential and form a massive black hole – making the model more or less identical to the black hole scenario?
It seems that not everybody does agree on the answer to this question. But Torres et al. (ref.26) may have the answer: in the boson-ball model all accreting stars are tidally disrupted. The atoms of the former star can move directly through the center of the boson-ball without being captured, following unbound orbits. The same goes for stellar black holes. And the result is, that no (massive) black hole is formed in the center. This disruption-mecanism is an importent difference in comparison to the neutrino-ball-scenario according to ref. 26.
However, no numerical simulations of this event (accretion of a star by a boson-ball) has been made yet (year 2000), so the solution of this problem seems not yet secured.

8. The Future
How can we distinguish the different models of the central mass in the Milky Way?
Several possibilities exist: very long baseline radio-interferometry VLBI (later also space-based instruments), infrared interferometry, X-ray-telescopes with improved angular resolution, and finally measurements of gravitational waves from stars orbiting the central mass.
We begin with the prospects of VLBI (ref.28). As can be calculated by a ray-tracing algoritm, it should be entirely possible to directly see the shadow of the black hole using short-wavelength radiowaves. In ref. 28 some calculated 'pictures' of this radioimage of the black hole in the center of the Milky Way is shown (fig. 7). In the calculations it is of course assumed that the black hole is surrounded by a source of radioemission – if not there is nothing to see! However, the source SgrA* is a strong radiosource, so that should not be a problem.

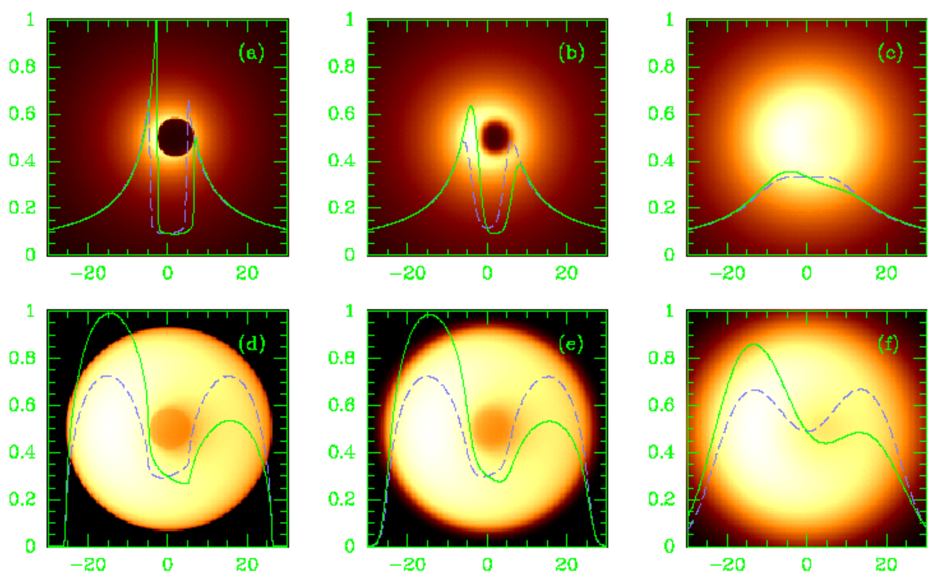

Fig. 7: calculated shadows of the black hole in the center of the Galaxy

The figures (a), (b) and (c) shows a black hole rotating at almost maximum rate, namely
$a = 0.998 \cdot \frac{1}{2} r_g$
where r_g is defined by eq. (3) - while the figures (d), (e) and (f) shows a nonrotating black hole, $a = 0$.
In the figures (a), (b) and (c)

the black hole is surrounded by an optically thin emitting gas with an emissivity proportional to r^{-2}, the gas being in free fall. The angle between the rotational-axes and the direction to the observer is 45°. The figures (d), (e) and (f) shows an emitting gas rotating in shells with the circular velocity in the equatorial-plane - having a uniform emissivity, viewing angle 45°. In these lower figures the emitting gas is limited to distances of $25 \cdot \frac{1}{2} r_g$ from the black hole (the scale on the horizontal axes is $\frac{1}{2} r_g$). The intensity-variations along the x-axes (the black hole being in (0,0)) are shown by the solid green curves while the intensity-variations along the y-axes are shown by the dashed purple curves.
The two left figures shows the result of the ray-tracing algoritm, while the two figures in the middle show what VLBI at 0.6 mm ideally would see, taking into account the interstellar scattering. The right figures shows what would be seen at the wavelength 1.3 mm.
The calculations take into account e.g. frame dragging, gravitational redshift, light bending and Doppler boosting.
The conclusion is that it is possible to see the shadow of the black hole using VLBI at a wavelength of 0.6 mm or shorter – and the size of the shadow will be approximately $10 \cdot \frac{1}{2} r_g$. This is close to the value calculated in eq. (22). The last measurement given in table 3 gives an intrinsic size of the central object of 6 r_g – however the uncertainty is of the same order – but we are here very close to the expected size of the dark shadow! So it is to be expected that we in the very near future will see, whether the predictions shown in fig.7 will be found in nature. Or we will see right through the center as expected in the two alternative scenarios mentioned above. No ray-tracing calculations like the ones shown in fig.7 using these models have however been carried out. Therefore the scattering-signature expected in these models cannot be direcly compared to the observations yet. But there will no doubt be a significant difference – making it possible to exclude either the black hole scenario or the other.
Takahashi (ref. 29) has calculated different forms for the shadow of the potential black hole in the center of the Milky Way – varying the rotational parameter, the viewing angle and using different forms of accretion-disks. The form of the shadow will depend on all these factors – making it difficult to determine e.g. the rotational parameter *a* from the shadow-form alone.

If we assume that the baseline *D* of the VLTI (in full operation in the year 2005) is 100 m, and that the wavelength λ is 2μm, we get an angular resolution of

(34) $$\theta = \frac{\lambda}{D} = \frac{2\mu\mathrm{m}}{100\,\mathrm{m}} = 2 \cdot 10^{-8} = 4\,\mathrm{mas}$$ VLTI-resolution

This corresponds to 32 r_g – hardly small enough to resolve a black disk of size 5 r_g.
Using VLT and the Large Binocular Telescope in interferometry mode we can measure the orbits of stars even weaker in NIR and closer to SgrA* than the star S2 – thereby pinpointing the position of SgrA* (assuming it to be identical to the IR-source) and getting even better constrains on the mass of the hole and – as a byproduct – measuring the distance to the galactic center even better. Also, for the stars orbiting close to the central mass it may be possible to measure periastron-shifts, the general relativistic shifts being prograde – as opposed to the contribution from a extended mass contribution which will be retrograde (ref. 30). Also, gravitational bending of light (here NIR) might be a way of probing the strong gravitational field near the central massive object.

If we turn our attension to the *X-ray telescopes*, the european XXM X-ray Observatory has a maximum angular resolution of 6 arcsec and the american Chandra X-ray Observatory a angular

resolution of 0.3 arcsec. Using the Chandra X-ray Observatory we will not be able to resolve a physical size of the central object within 33000 r_g .
However, if the proposed project X-ray observatory MAXIM is realised we will have an angular resolution of 0,001 mas(!!) and we will be able to see many details of accretion-disks and black disks of severel supermassive black holes, in the case of the black hole in the center of the Milky Way we will be able to see details as small as 0.1 r_g . It is expected that the X-ray-emission comes from the very inner parts of the accretion disk, making the radiation a sensible measure of the strong gravitational potential. The K-alfa line of iron at 6.4 keV is being redshifted, Doppler-broadned etc. The profile of this line is also a function of the rotational parameter, the line of inclination (the angle between the rotation-axes and the line of sight) – assuming a Kerr-black hole. Precission-measurements of this lineprofile is therefore a priority in e.g. the MAXIM X-ray mission (ref. 31).

Finally we will look at the possibilities for detecting *gravitational waves* from the galactic center. One might naively expect that all stars would be tidally disrupted before they entered a orbit so close to the galactic center that the emission af gravitational waves become important.
This is however not the case. If we take a look at a crude formula for the radius of tidal disruption of a star:

(35) $$r_{\text{tidal disruption}} = R_* \cdot \sqrt[3]{\frac{M}{M_*}}$$

where R_{star} is the radius of the star, M_{star} is the mass of the star and finally M is the mass of the black hole (the central mass), we see that what matters is actually only the mean-density of the star and the black hole mass. The bigger the mean-density, the smaller the tidal radius will be. Using typical values for e.g. for the radius and mass of white dwarfs or neutronstars and the mass of SgrA* we find that they are not tidally disrupted outside the event-horizon (a rough estimate). Stellar black holes will also pass the eventhorizon of the massive black hole without disruption. If we take into consideration more normal stars (main-sequence stars etc), modelcalculations show that the mean-density rises with smaller mass – reaching a maximum at about 0.07 M_{Sun} - at the transition to brown dwarfs (ref.19). And as it is expected that there are many low-mass-stars, we might expect that there are at least some of these stars much closer to the central mass of the Galaxy than the star S2.
In the weak field aproximation the orbit of a star can be treated as a Keplerian ellipse changing only slowly as the star looses energy to gravitational radiation.
The strain amplitude h_n (the amplitude of the relative change in length between the mirrors on the solid bodies defining the corners of the interferometer) of the of the gravitational waves (quadropole-type) belonging to frequency n times the orbital frequency $1/P$ - P being the period of the star in the orbit around the black hole - is (ref.19)

(36) $$h_n = \gamma(n,e) \cdot \frac{1}{D} \cdot \frac{G^2 M_{\text{BH}} \cdot M_*}{c^4 \cdot a}$$

where the factor $\gamma(n,e)$ is a function of the integer n and the orbital eccentrity e. D is the distance from the source to the observer, G is the gravitational constant, M_{BH} is the mass of the black hole, M_{star} is the mass of the star, a is the semimajor axes of the ellipse of the star.

The strain-amplitude of eq. (36) is the root mean square of strain-amplitudes for all possible directions of the orbit of the gravitational-wave-emitting star and averaged over the two possible polarizations.
In the case of a circular orbit only the n = 2 amplitude contributes. The gamma-factor depends on the polarization of the wave (2 possibilities) but is of the order 0.5.
If we as an example take a circular orbit, D = 8,0 kpc, $M_{\text{star}} = M_{\text{Sun}}$, $M_{\text{BH}} = 3.6 \cdot 10^6\, M_{\text{Sun}}$ and a = 0.1 mpc, we get

(37) $h_2 \approx \gamma(2{,}0) \cdot 10^{-20}$ $P = 0.05\,\text{yr}$ $\nu = \dfrac{1}{P} = 6.4 \cdot 10^{-5}\,\text{Hz}$

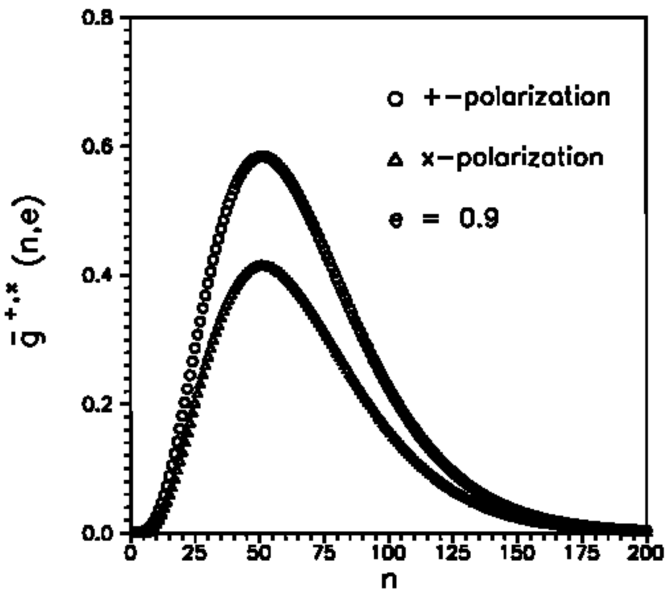

Fig. 8: gamma-factors of strain amplitude harmonics of an elliptic orbit (ref.33)

The period is calculated using Keplers 3. law.
The frequency ν is outside (below) the range of even the proposed spaceborn observatory LISA. But according to ref. 19 we might expect the orbits of the stars emitting gravitational radiation to be highly eccentric. In this case the hormonics with numbers far greater than 2 will dominate the frequency-spectrum, see e.g. ref. 32, 33. This is illustrated in fig.8 (ref.33), which shows the values of $\gamma(n,e)$ in the case e =0.9 for both polarizations of the gravitational wave. As can be seen on this figure, the maximum values of the γ-factors are still of order 0.5 – but now the higher harmonics are clearly dominating the frequency-spectrum – bringing some of the frequencies closer to the frequencies LISA can measure, namely (at least in the second generation LIGO-system) frequencies in the range 0.0001 Hz to 1 Hz at strain-amplitudes in the range of eq. (37). Values of e are expected to be much closer to 1 than in this illustrative example – making the higher harmonics even more important (ref.19).
Also, the value of the semimajor axes a might be smaller in the period before the star is tidally disrupted – making the strain-amplitude of eq. (36) bigger.
In a modelcalculation (ref.19) it is predicted that there might be 0.5 – 2 main-sequence stars with a mass below 0.6 solar masses emitting gravitational waves with a signal to noise-ratio (SNR) exceeding 10 (and 4 –8 with SNR greater than 3). White dwarfs and stellar black holes are less likely to be detected in gravitational waves by LISA in this modelcalculation.
The results are of course sensitive to the assumed initial mass-function and the evolution over time in this mass-function.
In fig.9 all the captures as a result of emission of gravitational radiation of the first 10 Gyr of the Monte Carlo simulation are shown giving their orbital parameters at the time of capture (when they plunge into into the massive black hole). The main-sequence-stars (MSS) are drawn with circles of

an area proportional to their mass. Also white dwarf (WD), neutron star (NS) and stellar black hole-captures are shown.
R_{peri} is the pericenter-distance, R_s is the Schwarzschild-radius of the central black hole, e is the eccentricity of the orbit.
Only stars with very elongated orbits are captured.

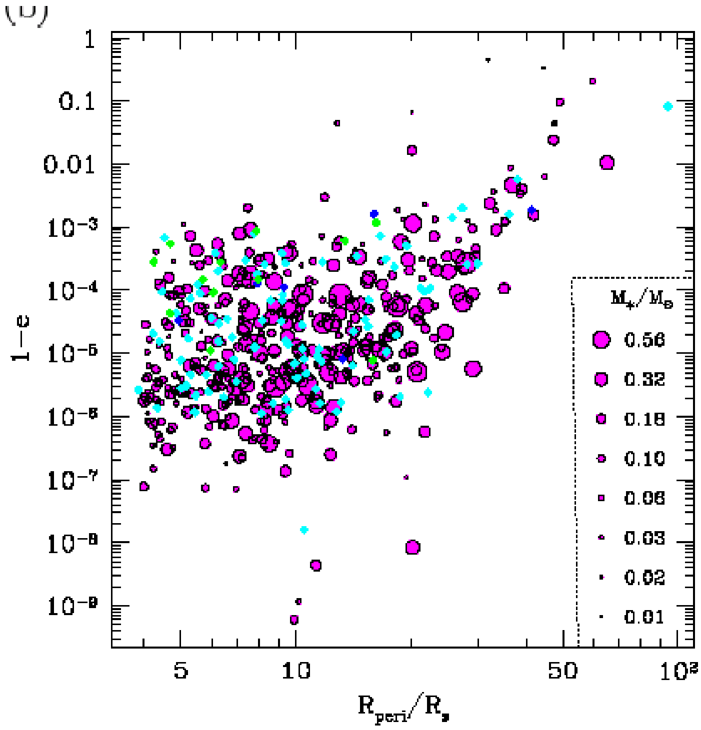

Fig. 9: orbital parameters at capture for MSS (purple), WD (cyan) NS (blue) and SBH's (green) (ref.19)

9. Conclusion

The massive object in the center of the Milky Way will be at the focus of a lot of research in the coming years. It is the best candidate known for a black hole – and we may in a few years time know whether the predictions of the General Theory of Relativity in the strong field regime are correct. If it is not a black hole, a few other possibilities have survived the observations. Some of these have been considered in this report – another interesting possibility is a socalled grava-star (*gra*vitational *va*cuum star). The event-horizon of the black hole is replaced by a transition-layer. In the central part of this model is matter obeying the equation of state $P = -\rho \cdot c^2$, where P is the pressure (negative!) and ρ is the mass-density, giving rise to a de Sitter Space – geometry (ref.34).
We will have to wait for some years before LISA can detect gravitational waves from stars orbiting the massive object and before MAXIM can give us very detailed pictures and spectra of the inner-most parts of the Milky Way center. But before this can happen, we might in the very near future see (or not see!) the black shadow of the exiting object in the midst of our Galaxy, using VLBI and mm-radiowaves or see the gravitational bending of light from stars very close to the center of the Galaxy – also making it possible to distinguish between different models of the massive central object.

So maybe – just maybe - the center of the Milky Way is even more strange than a black hole!? Soon we will know!

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Appendix 1 The Kerr-Newton-metric and related stuff

The Kerr-Newmann metric (in Boyer-Lindquist coordinates) is given by (see fx Misner, Thorne,Wheeler: Gravitation p.877 og 898)

$$-d\tau^2 = -\frac{\Delta}{\rho^2}\left[dt - a\sin^2\theta d\varphi\right]^2 + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)d\varphi - a dt\right]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

where

$$\Delta \equiv r^2 - \frac{2MG}{c^2}r + a^2$$

$$\rho^2 \equiv r^2 + a^2\cos^2\theta$$

$a \equiv S/(Mc)$=angular momentum per unit mass for the hole/c

From this the coefficients of the metric tensor can be read (the not listed values are 0):

$$g_{tt} = -\frac{\Delta - a^2\sin^2\theta}{\rho^2}, \qquad g_{t\varphi} = \frac{a\sin^2\theta(\Delta - (r^2+a^2))}{\rho^2}$$

$$g_{rr} = \frac{\rho^2}{\Delta}, \qquad g_{\varphi\varphi} = \frac{\sin^2\theta}{\rho^2}\left(\left(r^2+a^2\right) - \Delta a^2\sin^2\theta\right)$$

$$g_{\theta\theta} = \rho^2$$

The reciprocal tensor is therefore given by (not listed values are 0)

$$g^{tt} = -\frac{\left(r^2+a^2\right)^2 - \Delta a^2\sin^2\theta}{\rho^2\Delta} \qquad g^{t\varphi} = \frac{a\left(\Delta - \left(r^2+a^2\right)\right)}{\rho^2\Delta}$$

$$g^{rr} = \frac{\Delta}{\rho^2} \qquad g^{\varphi\varphi} = \frac{\Delta - a^2\sin^2\theta}{\rho^2\Delta\sin^2\theta} \qquad g^{\theta\theta} = \frac{1}{\rho^2}$$

From this we can calculate the Christoffel-indices we need:

$$\Gamma^r_{tt} = -½ g^{rr} \frac{\partial g_{tt}}{\partial r} \qquad \Gamma^r_{t\varphi} = -½ g^{rr} \frac{\partial g_{t\varphi}}{\partial r} \qquad \Gamma^r_{\varphi\varphi} = -½ g^{rr} \frac{\partial g_{\varphi\varphi}}{\partial r}$$

where

$$\frac{\partial g_{tt}}{\partial r} = \frac{\rho^2 - 2r^2}{\rho^4} \qquad \frac{\partial g_{t\varphi}}{\partial r} = -a \sin^2\theta \frac{\rho^2 - 2r^2}{\rho^4}$$

and

$$\frac{\partial g_{\varphi\varphi}}{\partial r} = \sin^2\theta \frac{2r\rho^4 + a^2 \sin^2\theta\left(\rho^2 - 2r^2\right)}{\rho^4}$$

The equation of motion for free fall is

(1) $$\frac{d^2 r}{d\tau^2} + \Gamma^r_{jk} \frac{dx^j}{d\tau}\frac{dx^k}{d\tau} = 0$$

But the second (and first) derivative of *r* in a circular motion is 0, and therefore the eq. of motion becomes

(1a) $$\Gamma^r_{\varphi\varphi}\omega^2 + 2\Gamma^r_{t\varphi}\omega + \Gamma^r_{tt} = 0$$

as postulated in eq. (11).
The equation (1) can be integrated to give a first order equation giving (see fx Hartle 2003 p.317 - 318 - we have here kept the proper time τ as parameter of the orbit)

(2) $$\frac{e^2 - 1}{2} = \frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + V_{\text{eff}}(r, e, l)$$

where

(3) $$V_{\text{eff}} = -\frac{GM}{r} + \frac{l^2 - a^2\left(e^2 - 1\right)}{2r^2} - \frac{M(l - ae)^2}{r^3}$$

is the effective potential governing the radial motion. Here the quantity *e* are the energy per unit mass of the particle, *l* is the angular momentum of the particle.
The criterion of stable circular orbits can be formulated as

(4a) $$\frac{e^2 - 1}{2} = V_{\text{eff}}(r, e, l)$$ because $\frac{dr}{d\tau} = 0$ (the *r*-velocity is 0)

(4b) $$\frac{\partial V_{\text{eff}}}{\partial r} = 0$$ because $\frac{d^2 r}{d\tau^2} = 0$ (*r*-acceleration is 0)

(4c) $\frac{\partial^2 V_{\text{eff}}}{\partial r^2} > 0$ stability against small variations in *r*

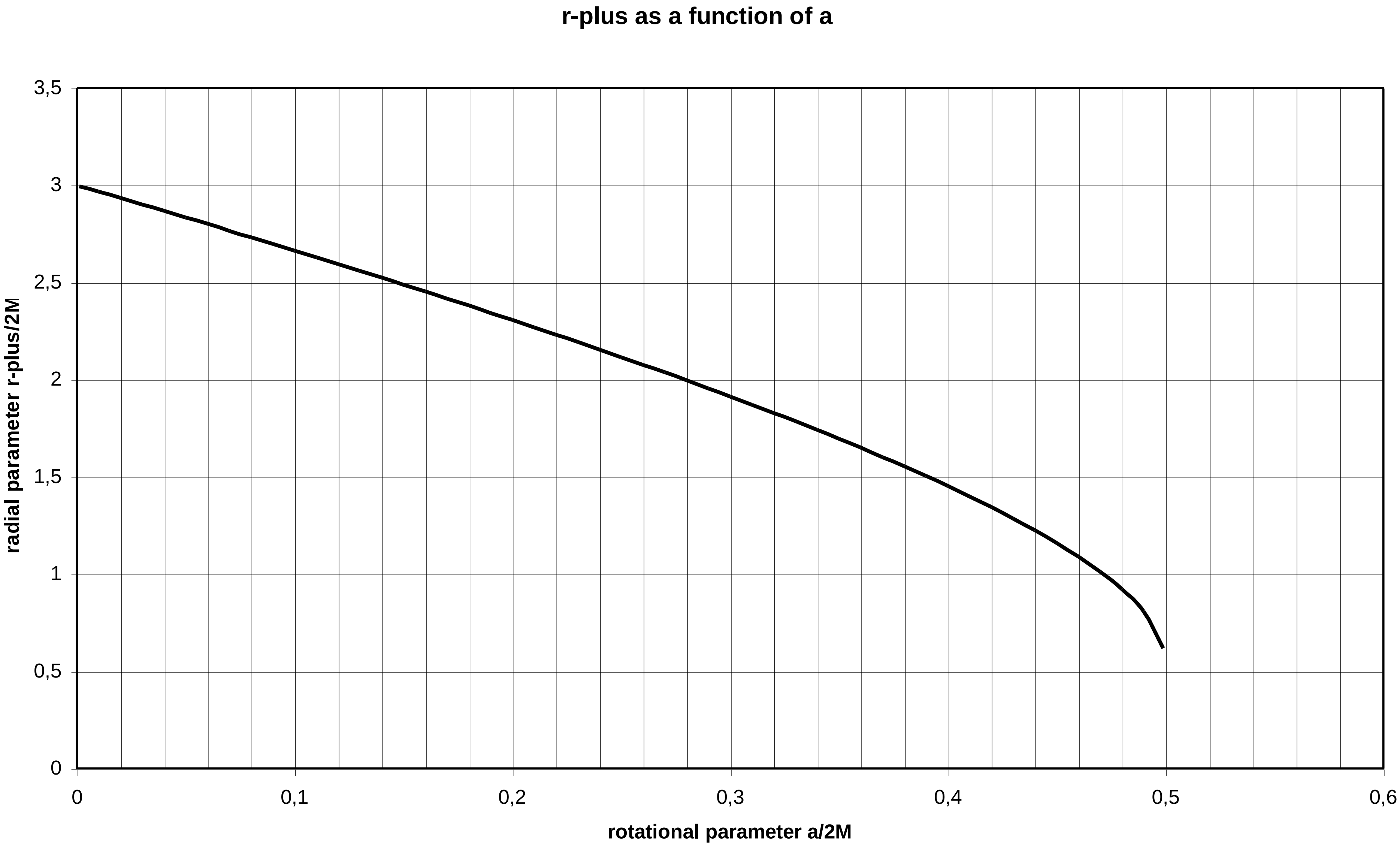
In the case of the innermost stable circular orbit the inequality sign in (4c) should be replaced by an equality-sign.
Thus we get 3 equations with the 3 unknown quantities *e*, *l*, *r* – all functions of *a*. It is not difficult to show that *r* has to be a solution of the equation

(5) $r = \sqrt{3r - \frac{a^2}{r}} - a\sqrt{3 - \frac{1}{r}}$ equation for r_{ISCO}

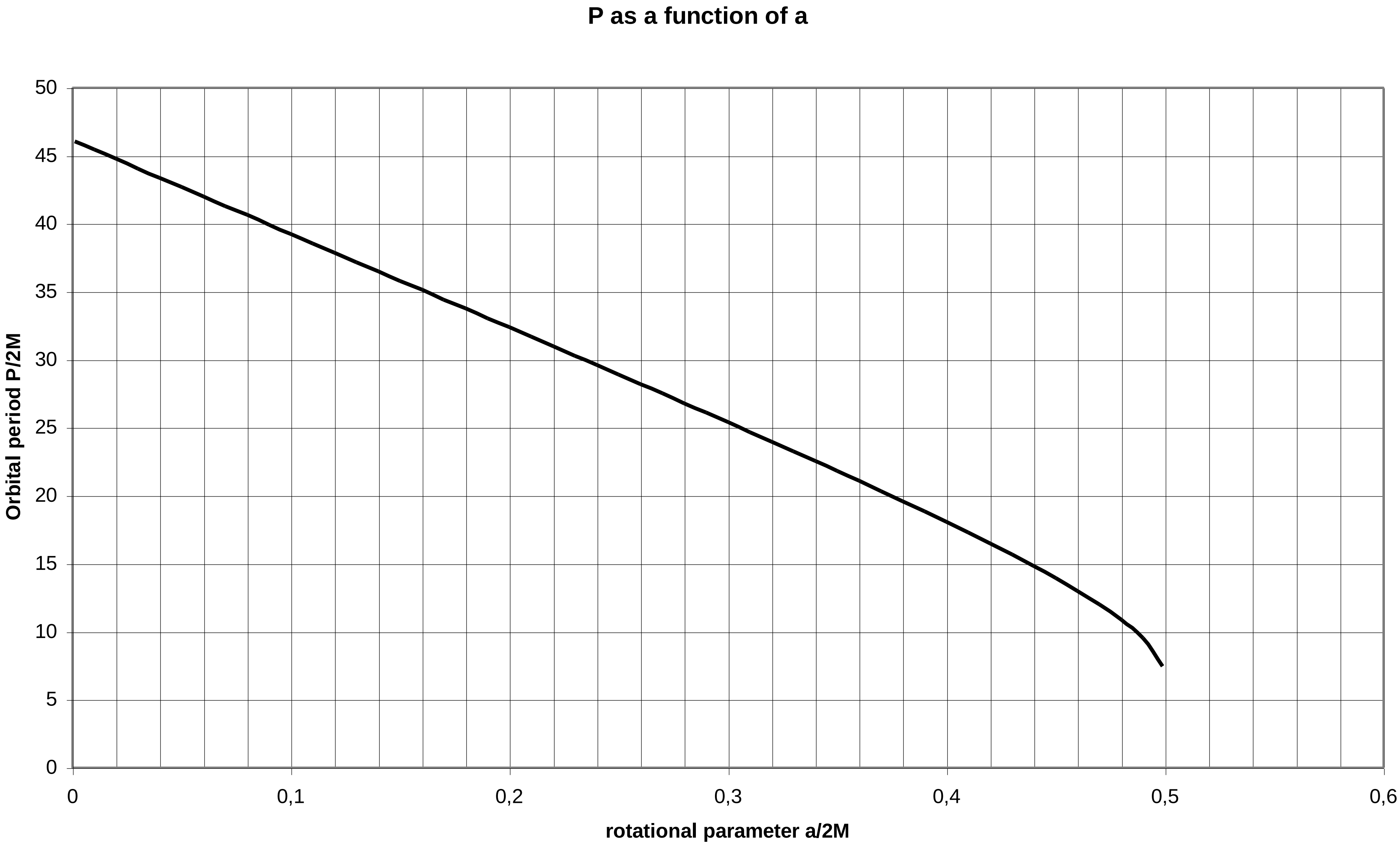
where *r* is measured in the unit r_g, a in the unit r_g.
The quantity r_g is defined by

(6) $r_g = \frac{2GM}{c^2}$

To get *r* as a function of *a* we must solve eq. (1a) taken together with (5). The result of this (numerical solution) can be found in appendix


r-plus as a function of a
radial parameter r-plus/2M
rotational parameter a/2M
3,5
3
2,5
2
1,5
1
0,5
0
0
0,1
0,2
0,3
0,4
0,5
0,6

Appendix 3


P as a function of a
Orbital period P/2M
50
45
40
35
30
25
20
15
10
5
0
0
0,1
0,2
0,3
0,4
0,5
0,6
rotational parameter a/2M